Math 54 Section 4: Quiz 4

Problem 1 True Or False?

(a) If a system $Ax = b$ has more than one solution, then so does the system $Ax = 0$.

(b) If the equation $Ax = 0$ has only the trivial solution, then A is row equivalent to the $n \times n$ identity matrix.

(c) If the columns of an $m \times n$ matrix A are linearly independent, then the columns of A span $\mathbb{R}^n$.

(d) If $BC = BD$, then $C = D$.

(e) If A and B are $n \times n$ matrices, then $(A + B)(A - B) = A^2 - B^2$.

(f) If A is invertible and $r \neq 0$, then $(rA)^{-1} = rA^{-1}$.

(g) $\det(AB - BA) = 0$ for any A and B.

(h) Any onto linear transformation is invertible.

(i) Row operations on a matrix can change the nullspace.

Problem 2 Suppose that v_1, v_2, v_3 are distinct points on a line L in $\mathbb{R}^3$. Here L does **not** necessarily pass through the origin. Show that $\{v_1, v_2, v_3\}$ is linearly dependent.

Problem 3 Suppose A is invertible. Explain why A^TA is also invertible. Then show that $A^{-1} = (A^TA)^{-1}A^T$.

Problem 4 Suppose that $A^n = 0$ for some $n > 1$. Find an inverse for $I - A$.

Problem 5 Compute the following determinants.

$$\begin{vmatrix} -1 & 5 & 2 \\ 5 & 6 & 3 \\ 1 & 3 & 1 \end{vmatrix} \qquad \begin{vmatrix} 9 & 5 & 2 \\ 1 & 0 & 0 \\ 4 & 4 & 1 \end{vmatrix}$$

Problem 6 Let A, B, C, D be $n \times n$ matrices with A invertible. Find matrices X and Y to produce the block factorization:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} I & 0 \\ X & I \end{bmatrix} \begin{bmatrix} A & B \\ 0 & Y \end{bmatrix}$$

and use this to show that:

$$\det \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \det(A) \det(D - CA^{-1}B)$$

Problem 7 Let V be a vector-space. We say that a subspace G is smaller than a subspace H of V if $G \subset H$, that is if G is a subspace of H. Let $v_1, \ldots, v_p$ be vectors in a vector-space V. What is the smallest subspace containing $v_1, \ldots, v_p$? Prove it.

Problem 8 The rank of a matrix A, denoted by $\text{rank}(A)$, is the number of pivots of A after row reduction. Equivalently, the rank is the dimension of the range/image of the linear map T_A of A, and the rank is also the dimension of the column space of A.

Let A be an $m \times n$ matrix and let B be an $n \times p$ matrix, such that $AB = 0$. Show that $\text{rank}(A) + \text{rank}(B) \leq n$.