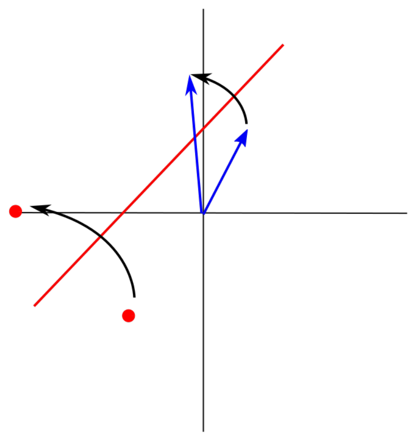
Math 54 Section 4: Quiz 1

Problem 1 (3 pts) For each of the following transformations $f : \mathbb{R}^n \to \mathbb{R}^m$, determine if f is linear or not linear. If not, give an example where linearity fails.

(a) $f : \mathbb{R}^3 \to \mathbb{R}^3$ given by $f(x, y, z) = (x + y, y + z, z + x)$.

(b) $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = |x| + y$.

(c) $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by reflection across a line that does **not** go through $\mathbf{0}$. See the picture below.



Problem 2 (1.9 # 19) Verify that the transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by:

$$T(\mathbf{v}) = T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3)$$

is linear by finding a matrix that implements the map. Here $\mathbf{v} = (x_1, x_2, x_3)$. Is T onto and/or one-to-one?

Problem 3 (2.1 # 11) Let:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \qquad D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Compute AD and DA. Find a 3×3 matrix B that is not the identity such that $AB = BA$.

Problem 4 (2.1 # 25) Suppose A is an $m \times n$ matrix and there exists $n \times m$ matrices C and D such that $CA = I_n$ and $AD = I_m$. Prove that $m = n$ and $C = D$. (Hint: Consider the product CAD).

Problem 5 Suppose that $AD = I_m$ (where A is $m \times n$, D is $n \times m$ and I_m is the $m \times m$ identity matrix). Show that the linear map $T : \mathbb{R}^n \to \mathbb{R}^m$ given by $T(x) = Ax$ is onto.