Open Problems for the Barbados Graph Theory Workshop 2017

An up-to-date version of this list is maintained at `https://web.math.princeton.edu/~gjg2/barbadosopenproblems2017.pdf`.

Please send your open problems and comments to Gregory Gauthier (gjg2@princeton.edu) to be included here.

The website for the workshop is `https://web.math.princeton.edu/~pds/barbados17/index.html`.

1. Let a, b be vertices of a graph G, such that for every vertex v, there is an induced path between a, b containing v. Can we test in polynomial time whether all induced paths between a, b have the same length? *(Alex Scott and Paul Seymour)*

2.

A graph G is **k-colourable with defect** d, or simply **(k, d)-colourable**, if each vertex v of G can be assigned one of k colours so that at most d neighbours of v are assigned the same colour as v. That is, each monochromatic subgraph has maximum degree at most d. Let's focus on minimising the number of colours k rather than the degree bound d. This viewpoint motivates the following definition [7]. The *defective chromatic number* of a graph class $\mathcal{C}$ is the minimum integer k (if such a k exists) for which there exists an integer d such that every graph in $\mathcal{C}$ is (k, d)-colourable.

Consider the following three examples: Archdeacon [2] proved that for every surface Σ, the defective chromatic number of graphs embeddable in Σ equals 3. Edwards, Kang, Kim, Oum, and Seymour [3] proved that the class of graphs containing no K_{s+1} minor has defective chromatic number s (which is a weakening of Hadwiger's conjecture). Ossona de Mendez, Oum, and Wood [7] proved that for $s \leq t$, the class of graphs containing no $K^*_{s,t}$ minor has defective chromatic number s (which implies the above two results). Ossona de Mendez et al. [7] conjectured the following:

Conjecture [7]: For every connected graph H, the defective chromatic number of H-minor-free graphs equals the tree-depth of H minus 1.

Here the **tree-depth** of a connected graph H is the minimum height (measured in number of vertices) of a rooted tree T such that H is a subgraph of the closure of T. Here the **closure** of T is obtained from T by adding an edge between every ancestor and descendent in T. The **height** of a rooted tree is the maximum number of vertices on a root-to-leaf path. The **tree-depth** of a disconnected graph H is the maximum tree-depth of the connected components of H.

Ossona de Mendez et al. [7] showed that the defective chromatic number of H-minor-free graphs is at least the tree-depth of H minus 1. The construction is from [3]. Let $G(s, d)$ be the closure of the complete d-ary tree of height s. We prove by induction on s that every $(s-1)$-colouring of $G(s, d)$ has a vertex of monochromatic degree at least d. In the base case, since $G(2, d)$ is the star $K_{1,d}$, every 1-colouring of $G(2, d)$ has a vertex with monochromatic degree d, as claimed. Consider an $(s-1)$-colouring of $G(s, d)$. Let v be the dominant vertex in $G(s, d)$. Say v is coloured blue. Then $G(s, d) - v$ consists of d disjoint copies of $G(s-1, d)$. If some copy of $G(s-1, d)$ contains no blue vertex, then $G(s-1, d)$ is $(s-2)$-coloured, and by induction contains a vertex with monochromatic degree at least

d, and we are done. Otherwise, each copy of $G(s-1,d)$ contains a blue vertex, in which case, v has monochromatic degree at least d.

Now, given a connected graph H, let $s+1$ be the tree-depth of H. By definition, $G(s,d)$ has tree-depth s. Since tree-depth is minor-monotone, every minor of $G(s,d)$ has tree-depth at most s. Thus H is not a minor of $G(s,d)$. As proved above, every $(s-1)$-colouring of $G(s,d)$ has a vertex with monochromatic degree at least d (which is unbounded). Thus the defective chromatic number of the class of H-minor-free graphs is at least s.

The conjecture is true for $H = K_t$ ([3]), for $H = K_{s,t}$ or $H = K^*_{s,t}$ (where $K^*_{s,t}$ is formed by taking K_s, subdividing each edge once, and t edges all adjacent to the s branch vertices) ([7]), and for connected graphs with tree-depth at most 3 ([7]), although the tree-depth 4 case is non-trivial and interesting. Mohar, Reed, and Wood [6] proved that the defective chromatic number of the class of graphs with circumference k (that is, C_{k+1}-minor-free graphs) is at most $3\log_2 k$, which is within a factor of 3 of the conjecture. This also provides the best known result when H is a path.

Sergey Norin, Alex Scott, and Paul Seymour showed defective chi-boundedness:

Theorem 1 (Norin, Scott, Seymour)**.** *For all t there exists a constant $f(t)$ such that, for every connected graph H with tree-depth t, there is a constant $g(H)$ such that H-minor free graphs can be colored in $f(t)$ colors such that monochromatic components have size at most $g(H)$.*

The function f satisfies the recurrence $f(t) \leq 4f(t-1)+2$, giving that $f(t)$ is $O\left(4^t\right)$.

(David Wood)

3. (a) Does every cubic bridgeless graph with n vertices contain a closed walk of length at most $5n/4$ that visits each vertex? The positive answer would improve the known approximation ratios of TSP algorithms for cubic graphs. On the other hand, constructions found by Lukotka and Mazak, and by Dvorak, Mohar and myself show that this bound would be tight up to an additive factor.

   (b) Given a bridgeless graph G, consider the problem of finding a collection of cycles $C_1, \ldots, C_k$ covering all edges of G such that $\sum_{i=1}^k |C_k|$ is minimized. Let $m = |E(G)|$. It is known that an upper bound for the sum is $\frac{5m}{3}$, and if G is cubic, then the upper bound can be improved to $\frac{8m}{5}$. Generally, $\frac{7m}{5}$ is a lower bound; if true, it would imply the cycle double cover conjecture.

   (Dan Král')

4. Prove that in a random bipartite graph with n vertices on each side of the bipartition, the number of perfect matchings is $0 \mod 3$ with probability $1/3$ asymptotically. *(Stéphan Thomassé)*

5. A vertex coloring of a graph G is **nonrepetitive** if for every path with even number of vertices, the color sequence in the first half is different from the color sequence of the second half. This is a strengthening of the usual proper coloring, and also star coloring. We let $\pi(G)$ denote the nonrepetitive chromatic number of G; as every nonrepetitive coloring is a proper coloring, we have $\chi(G) \leq \pi(G)$. There exist bipartite graphs with arbitrarily large nonrepetitive chromatic number.

   An inclusion-exclusion argument shows that the number of nonrepetitive k-colorings of a graph is a polynomial in k.

   (a) Is there a simpler argument to prove that it is indeed a polynomial?

   (b) The usual deletion-contraction doesn't work. Can something be salvaged?

(c) Compute the nonrepetitive chromatic polynomial of paths. This should imply Thue's 1906 theorem that every path can be nonrepetitively 3-colored.

(d) Compute the nonrepetitive chromatic polynomial of trees. This should imply the known fact that every tree can be nonrepetitively 4-colored.

(e) One of the main open problems about nonrepetitive coloring is whether there is an absolute constant c such that every planar graph can be nonrepetitively c-colored. Following Birkhoff, what can we say about the roots of the nonrepetitive chromatic polynomial of planar graphs?

(Vaidy Sivaraman)

6. Let $f(k)$ be the largest n such that the edges of K_n can be k-coloured so that each colour gives a triangle-free graph. So $f(1) = 2$, $f(2) = 5$, and in general $f(k) \leq kf(k-1) + 1$ (because otherwise some vertex would have more than $f(k-1)$ neighbours in the same colour). It turns out that $f(3) = 16$, so equality holds in this bound for $k = 2, 3$. Does equality hold for all k? *(Eli Berger, Maria Chudnovsky, Paul Seymour, Sophie Spirkl)*

Comments:

1. This is just asking for the value of the Ramsey number $R(3, \ldots, 3)$, isn't it?

   I think this is a hard problem: it's an old Erdős problem even to find the limit of $f(k)^{1/k}$. And it's known that equality does not hold for your recurrence: the bound for $k = 4$ would be 65 (corresponding to a Ramsey number of 66). But the argument in this paper suggests that it should be at most 61: `http://www.public.iastate.edu/~ricardo/r3333global.pdf` There is more relevant history here: `https://www.cs.rit.edu/~spr/PUBL/sur14.pdf` *(Alex Scott)*

7. (a) Prove that the tree width of an even-hole-free graph G is bounded by a function in the clique number of G.

   (b) Solve Problem (a) for triangle-free graphs.

   (c) Is there a polynomial-time algorithm for the Minimum Weighted Coloring Problem for graphs with stability number 2?

   (d) Prove the Erdős-Hajnal conjecture for odd-hole-free graphs.

   (e) Prove that G is 2-divisible if and only if G is odd-hole-free.

   (f) Prove that G is perfectly divisible if and only if G is odd-hole-free.

   (g) Prove that there exists $\varepsilon > 0$ such that if G is an n-vertex graph that is P_5-free and C_5-free, then $\omega(G) \geq n^{\varepsilon}$ or $\alpha(G) \geq n^{\varepsilon}$.

   (h) Prove that G is perfectly divisible if G is P_5-free and C_5-free.

   Definitions and background: A hole is a chordless cycle with at least four vertices. A hole is odd (even) if it has an odd (even) number of vertices. A graph is odd-hole-free (even-hole-free) if it does not contain an odd hole (even hole) as an induced subgraph. The clique number $\omega(G)$ of a graph is the number of vertices in a largest clique of G. The stability number $\alpha(G)$ is the clique number of the complement $\overline{G}$ of G.

   If Problem (a) is solved in the affirmative, then for every fixed k, there is a polynomial-time algorithm to k-color an even-hole-free graph.

   A weighted graph G is a graph where each vertex v is given a weight (integer) $w(v)$. The Minimum Weighted Coloring Problem asks for stable sets $S_1, \ldots, S_r$ where each S_i is assigned a weight (non-negative integer) $I(S_i)$ such that (i) for each vertex v, $w(v) \leq \sum_{v \in S_i} I(S_i)$, and (ii) the sum $I(S_1) + \ldots + I(S_r)$ is minimized.

One might try to solve Problem (c) by substituting each vertex v by a clique with $w(v)$ vertices. But the size of the "blown up" graph is $O(W+n)$, where W is the sum of all $w(v)$, where as the size of the problem is $O(n+\log W)$.

The Erdős-Hajnal Conjecture states that for any fixed graph H, if a graph G is H-free, then $\omega(G) > n^\varepsilon$ or $\alpha(G) > n^\varepsilon$ for some $\varepsilon = \varepsilon(H)$.

A graph G is 2-divisible if for every induced subgraph H of G with at least one edge, there is a partition (A, B) of $V(H)$ such that $\omega(A) < \omega(H)$ and $\omega(B) < \omega(H)$. A proof of the conjecture stated in (e) would imply $\chi(G) \leq 2^{\omega(G)}$ for odd-hole-free graphs.

A graph G is perfectly divisible if and only if there exists a partition $\left(A_1, \dots, A_{\omega(G)}\right)$ such that for each i, $G[A_i]$ is perfect. An affirmative answer to the conjecture in (f) would prove the Erdős-Hajnal Conjecture for odd-hole-free graphs with $\varepsilon = \frac{1}{3}$.

For problem (b), Maria Chudnovsky and Paul Seymour note that the graph formed by taking C_9 and adding a vertex adjacent to every third vertex is triangle-free, has no even holes, and has a theta subgraph. This forecloses the approach of eliminating thetas to bound treewidth.

Maria Chudnovsky proved that if G has no C_5 and no P_5, then G is 2-divisible. Chudnovsky and Vaidy Sivaraman proved that if G has no bull and no odd holes, then G is perfectly divisible. They also proved that if G has no bull and no P_5, then G is perfectly divisible. *(Chính Hoàng)*

8. Is it true that every even-hole free graph (that isn't empty) contains an edge whose removal does not create an even hole?

   The problem is motivated by a similar problem for triangle-free graphs with no hole of length divisible by 3, which would imply the 3-colorability of those graphs.

   Paul Seymour notes that a similar conjecture using C_4-free graphs instead of even-hole-free graphs is false, as demonstrated by the icosahedron.

   Cornuejols conjectured that if G is a bipartite graph with no holes of length $2 \pmod 4$, then there is an edge that can be deleted to produce a graph with no holes of length $2 \pmod 4$. *(Paul Wollan; contributed by Marthe Bonamy)*

9. A **Clique-stable set separator** of a graph G is a family F of bipartitions of $V(G)$ such that, for every clique K and stable set S of G, either $K \cap S \neq \emptyset$ or there exists a cut (B, W) in F that separates K and S, meaning that $K \subseteq B$ and $S \subseteq W$. Note that every graph with n vertices has a clique-stable set separator of size $n^{O(\log n)}$. A class C of graphs satisfies the **polynomial CS-separation property** if there exists a polynomial P such that every G in C admits a CS-separator containing at most $P(|V(G)|)$ cuts. Not every graph has the polynomial CS-separation property, due to E Göös 2015.

   Open question by Yannakakis (1991): does the class of perfect graphs satisfy the polynomial CS-Separation property? A positive answer to this question is a necessary condition for the existence of a compact extended formulation for the stable set polytope in perfect graphs.

   Chordal graphs have the polynomial (even linear) CS-Separation property because they have a linear number of maximal cliques. A graph is weakly chordal if G contains no hole and antihole of length at least 5.

   F. Maffray asked whether the class of weakly chordal graphs satisfies the polynomial CS-Separation property. The answer is yes, due to two results:

   Theorem 2 (Bonamy, Bousquet, Thomassé)**.** *Let k be an integer and let $\mathcal{C}_k$ be the class of graphs with no hole of length at least k and no anti hole of length at least k. Then $\mathcal{C}_k$ satisfies the Strong Erdős-Hajnal property, meaning there exists $c > 0$ such that every graph* *in $\mathcal{C}_k$ contains two disjoint sets of vertices V_1, V_2 of size at least $c|V(G)|$, such that V_1 is complete or anticomplete to V_2.*

Theorem 3 (Bousquet, Lagoutte, Thomassé)**.** *If $\mathcal{C}$ is a hereditary class satisfying the Strong Erdős-Hajnal property, then $\mathcal{C}$ satisfies the polynomial CS-separation property.*

(Aurélie Lagoutte)

10. Consider the minor containment relation. Then:

    - $\{K_3, K_{1,3}\}$-free = linear forest (disjoint union of paths)
    - $\{K_4, K_{2,3}\}$-free = outerplanar
    - $\{K_5, K_{3,3}\}$-free = planar
    - $\{K_6, K_{3,4}\}$-free = ?

    There have been papers on K_6-free graphs, most notably the paper by Robertson, Seymour and Thomas on Hadwiger's conjecture for $k = 6$. Also, there are some results on $K_{3,4}$-free graphs. If you forbid both, maybe something nice happens? A nice structure theorem?

    I was led to this when looking at the Colin de Verdière invariant μ:

    - $\{K_3, K_{1,3}\}$-free = graphs with $\mu \leq 1$
    - $\{K_4, K_{2,3}\}$-free = graphs with $\mu \leq 2$
    - $\{K_5, K_{3,3}\}$-free = graphs with $\mu \leq 3$
    - $\{K_6, K_{3,4}\}$-free - Pattern breaks

    Results by Robertson, Seymour and Thomas and by Lovász and Schrijver imply that: Petersen family-free = linklessly embeddable = graphs with $\mu \leq 4$. *(Vaidy Sivaraman)*

11. A matching in a hypergraph H is a set of disjoint edges and a cover in H is a set of vertices meeting all edges. Let $\nu(H)$ be the maximal size of a matching in H and $\tau(H)$ the minimal size of a cover.

    An old conjecture of Tuza [8] is that given any graph G the minimal number of edges needed to cover all the triangles in G is at most 2 times the maximal number of edge-disjoint triangles in G. In other words, Tuza's conjecture is $\tau(T_G) \leq 2\nu(T_G)$, where T_G is the 3-uniform hypergraph with vertex set $E(G)$ and whose edges are triples in $E(G)$ that form a triangle. The best known bound is $\tau \leq 2.86\nu$ [5]. The only known tight examples for which $0 < \tau(T_G) = 2\nu(T_G)$ are when G is K_4 or K_5.

    We suggest the following generalization of Tuza's conjecture. For a 3-uniform hypergraph $H = (V, E)$ let $H^{(2)}$ be the **pair hypergraph** of H, namely the 3-uniform hypergraph whose vertex set is $\binom{V}{2}$ and whose edge set is $\{\binom{e}{2} \mid e \in E\}$. It turns out that the family of pair hypergraphs has many forbidden structures (for example, a pair hypergraph cannot contain a copy of the Fano plane).

    We conjecture that Tuza's conjecture is true for every pair hypergraph, namely, that in every 3-uniform hypergraph H we have $\tau(H^{(2)}) \leq 2\nu(H^{(2)})$ (Tuza's conjecture is the special case where H is the hypergraph of triangles in G). See [1] for many more details on this problem. *(Shira Zerbib)*

12. Let a family $\{S_i\}$ of tournaments be defined as follows. S_1 is the tournament on 1 vertex. For $i > 1$, S_i is obtained by blowing up two vertices of the cyclic triangle into two copies of S_{i-1}. Prove that for every integer $i > 1$, there exists $f(i)$ such that every tournament T with domination number at least $f(i)$ contains an isomorphic copy of S_i.

    This problem is taken from [4].

Here, a tournament is a complete oriented graph. The domination number of a tournament T is the smallest size of a subset S of the vertices of T so that every vertex in $V(T) \setminus S$ has an in-neighbour in S. The chromatic number of a tournament is the minimum number of sets into which $V(T)$ can be partitioned such that each set induces a transitive tournament. *(Anita Liebenau)*

13. Let $f(t)$ denote the maximum chromatic number of K_t-minor-free, triangle-free graphs. What is the asymptotic behaviour of $f(t)$? In particular, is $f(t) < t$ for large enough t, i.e. does the Hadwiger's conjecture hold for triangle-free graphs for large t?

    Some background:

    Kühn and Osthus proved that Hadwiger's conjecture holds for graphs of girth at least 5.

    Combining results of Kostochka and Thomason on density of K_t- minor-free graphs with the theorem of Shearer on independence number of triangle-free graphs, one can deduce that every K_t-minor-free, triangle- free graph G on n vertices satisfies $\alpha(G) \geq cn\sqrt{\log t}/t$ for some absolute constant c. Thus at least the independence number is large enough.

    The best lower bound on $f(t)$ that I know of is on the order of $t^{2/3}$ up to a polylog factor and comes from a random construction. *(Sergey Norin)*

14. A graph G is k-**bend** if it can be represented on some underlying grid as follows. Each vertex corresponds to a path in the grid (no restrictions on the whereabouts of the endpoints of these paths); each path takes at most k turns; and vw is an edge of G if and only if the paths representing v and w intersect in some non-trivial interval (not just a point). Every graph is k-bend for some k.

    The clique chromatic number of a 0-bend graph is at most 2 (because 0-bend graphs are interval graphs). The clique chromatic number of a 1-bend graph is at most 4 (Bonomo, Mazzoleni and Stein). Can this bound be lowered to 3 (that would be best possible, because C_5 is 1-bend and 3-clique-chromatic)? How about k-bend graphs in general, is their clique chromatic number bounded by some function of k? *(Maya Stein)*

15. Given an ordering $v_1, \ldots, v_n$ of the vertices of a graph G, we say that v_i r-**strongly reaches** v_j if $j \leq i$ and there exists a v_i-v_j path of length at most r such that all internal vertices are to the right of v_i in the ordering. The r-strong coloring number of G is the minimum k such that there exists an ordering where each vertex r-strongly reaches at most k vertices. Note the 1-strong coloring number of G is the degeneracy number.

    Open problem: Suppose that a class of graphs $\mathcal{C}$ is such that there exists $\varepsilon > 0$ such that graphs in $\mathcal{C}$ have separators of size $O\left(n^{1-\varepsilon}\right)$. Does this imply that there is a fixed constant c such that graphs in C have r-strong coloring numbers $O\left(r^c\right)$?

    The converse is known to be true. *(Sergey Norin)*

    Given a graph G with n vertices, we say that S is a separator if every component of $G \setminus S$ has at most $\frac{2n}{3}$ vertices. We are interested in classes of graphs, closed under taking subgraphs, with separators of sublinear size; that is, there exists $0 < \delta$ such that every graph G in the class has a separator S of size $O\left(n^{1-\delta}\right)$.

    Let $\mathcal{C}$ be a class of graphs closed under taking subgraphs. A result of Dvorak and Norin states that there exists $\delta > 0$ such that graphs in $\mathcal{C}$ have $O\left(n^{1-\delta}\right)$ separators if and only if there exists a constant $c \geq 0$ every graph G in $\mathcal{C}$, the average degree of an r-shallow minor of G is $O(r^c)$.

    We say that H is an r-shallow minor of G if H can be formed from a subgraph of G by contracting edges that induce graphs with each component having radius at most r. *(Gwenaël Joret, David Wood)*

16. Conjecture ([9]): For all k, there exists a function $f(k)$ such that for all graphs G, if the connectivity $\kappa(G) \geq f(k)$, then there exists a spanning, bipartite subgraph $H \subseteq G$ such that $\kappa(H) \geq k$.

    This conjecture holds up to a $\log n$ factor [10]. *(Thomassen; contributed by Michelle Delcourt)*

17. A random d-regular graph is typically d-vertex-connected. For all k, does there exists a function $f(k)$ such that an $f(k)$-regular random graph G has a spanning, bipartite subgraph $H \subseteq G$ such that $\kappa(H) \geq k$ asymptotically almost surely? *(Michelle Delcourt)*

    Solution *(Sergey Norin)* with $f(k) = 2k-1$. Note that if G has n vertices and $d \geq 3$, almost surely every set X with $|X| \leq c(d)$, then X induces at most $|X|$ edges and almost surely if $|X| \leq \frac{n}{2}$ then X has at least $\frac{1}{10}|X|$ edges going to $V(G) \setminus X$.

    Take any maximum cut, and suppose that the corresponding graph is not k-connected. Then, there is a set of size at most $k-1$ separating a component of size at most $\frac{n}{2}$. Switch sides of the cut for X. Consider the case $|X|$ is bounded. Let S be a cutset of size $k-1$, and let $Z = (V(G) \setminus X) \setminus S$.

    Let α be the number of edges with both ends in X. There are at most $k-1+|X|-\alpha$ from X to S. and there are at least $|X|d - k + 1 - |X| - \alpha$ edges from X to Z. After switching edges, we gain the edges from X to Z and lose the edges from X to S. Since we had a maximum cut, we have $|X|d \leq 2(k-1+|X|)$. So $|X|(2k-1) \leq 2(k-1+|X|)$. But this is false when $|X| \geq 3$, because $3(2k-1) > 2(k-1+3)$ so $6k-3 > 2k+4$. The $|X| \leq 2$ case can be solved as well.

18. We say G is Ramsey for H if for every 2-coloring of $E(G)$ there exists a monochromatic (not necessarily induced) copy of H. We say that H_1 and H_2 are Ramsey equivalent if for every G, we have G is Ramsey for H_1 if and only if G is Ramsey for H_2.

    Question: can you find nonisomorphic trees T_1 and T_2 such that T_1 is a Ramsey equivalent of T_2?

    What we know: K_k is Ramsey equivalent to K_k disjoint union some smaller cliques (at most one K_{k-1}), but K_k is not Ramsey equivalent to K_k minus an edge. Also, paths and stars are not Ramsey equivalent to any other nonisomorphic graph.

    What we don't know: are there any connected nonisomorphic Ramsey equivalent graphs? *(Anita Liebenau)*

19. Let $\mathcal{A}$ be a family of subsets of $[n]$. We say that $\mathcal{A}$ is intersecting if $A \cap B \neq \emptyset$ for all $A, B \in \mathcal{A}$. It is easy to see that $|\mathcal{A}| \leq 2^{n-1}$ and that any intersecting family can be extended to a family of size 2^{n-1}.

    We say that $\mathcal{A}$ is 3-matching-free if for every distinct $A, B, C \in \mathcal{A}$, A, B, C are not all pairwise disjoint. It is easy to see that we can find a 3-matching-free family of size $\frac{3}{4}2^n$ (fix two points and consider all sets with at least one of those two points), and a straightforward argument shows that in every maximal 3-matching-free family $\mathcal{A}$, either A or $\bar{A}$ is in $\mathcal{A}$, giving a lower bound of $\frac{1}{2}2^n$ (and one can construct a maximal 3-matching-free family of size $\frac{1}{2}2^n + 1$ by adding $\emptyset$ to any intersecting family containing $[n]$). It is unknown whether a better lower bound for the size of a maximal 3-matching-free family not containing $\emptyset$ exists. *(Peter Keevash)*

20. For fixed t and $k \gg t$, what is the maximum number $p_t(k)$ of t-covers in a k-uniform intersecting hypergraph H with $\tau(H) = t$?

    Here $\tau(H)$ is the minimum size of a vertex cover of H.

The problem is motivated by the following question: what is the maximum number of edges in a k-uniform intersecting hypergraph on $[n]$ with $\tau(H) = t$? The case $t = 1$ is the Erdős-Ko-Rado theorem. When $t = 2$, the optimal construction consists of picking a special edge e and vertex $x \notin e$, and taking every edge containing x, exactly one vertex of e, and $k-2$ vertices elsewhere. This gives $\binom{n-1}{k-1} - \binom{n-k-1}{k-1} + 1$ edges.

Solving the first problem gives an answer to the question: taking H to be k-uniform, $\tau(H) = t-1$, intersecting, number of $t-1$ covers in H is maximum. The reduction is due to Frankl in the case $t = 3$, and in the case $t = 4$ and $t = 5$ by Frankl and Ota and Tokushige.

Conjecture (Frankl): $p_t(k) = k^t - \binom{t}{2}k^{t-1} + O(k^{t-2})$. *(Liana Yepremyan)*

References

[1] Ron Aharoni and Shira Zerbib. m-matchings and m-covers. 2016. arXiv: 1611.07497.

[2] Dan Archdeacon. A note on defective colorings of graphs in surfaces. *J. Graph Theory*, 11(4):517–519, 1987.

[3] Katherine Edwards, Dong Yeap Kang, Jaehoon Kim, Sang-il Oum, and Paul Seymour. A relative of Hadwiger's conjecture. *SIAM J. Discrete Math.*, 29(4):2385–2388, 2015.

[4] Ararat Harutyunyan, Tien-Nam Le, Stéphan Thomassé, and Hehui Wu. Coloring tournaments: from local to global. arXiv: 1702.01607.

[5] Penny E. Haxell. Packing and covering triangles in graphs. *Discrete Math.* 195(1):251–254, 1999.

[6] Bojan Mohar, Bruce Reed, and David R. Wood. Colourings with bounded monochromatic components in graphs of given circumference. 2016. arXiv: 1612.05674.

[7] Patrice Ossona de Mendez, Sang-il Oum, and David R. Wood. Defective colouring of graphs excluding a subgraph or minor. 2016. arXiv: 1611.09060.

[8] Zsolt Tuza. A Conjecture. *Finite and Infinite Sets*, Eger, Hungary 1981, A. Hajnal, L. Lovász, V.T. Sós (Eds.), *Proc. Colloq. Math. Soc. J. Bolyai*, 37, North-Holland, Amsterdam, 1984.

[9] Carsten Thomassen. Configurations in graphs of large minimum degree, connectivity, or chromatic number. *Annals of the New York Academy of Sciences*, 555.1: 402–412, 1989.

[10] Michelle Delcourt and Asaf Ferber. On a Conjecture of Thomassen. *Electronic Journal of Combinatorics*, 22, 3.2: 1–8, 2015.