MAT 215: PROBLEM SET 8

DUE THURSDAY APRIL 7

Reading: Abbot, Sections 7.5, 6.2-6.4

Problem 1: Let $f\colon (0,1] \to \mathbb{R}$ be a function that is Riemann integrable on $[c,1]$ for each $c \in (0,1]$. Define

$$\int_0^1 f(x)dx = \lim_{c\to 0} \int_c^1 f(x)dx,$$

when the limit exists.

(i) Show that when f is integrable on $[0,1]$, the limit definition and the definition of the integral on $[0,1]$ agree.

(ii) Show that $f(x) = 1/x^{1/2}$ is integrable on $[0,1]$ under the new definition.

Problem 2: Suppose f is integrable on $[1,a]$ for every $a \in (1,\infty)$. Define

$$\int_1^\infty f(x)dx = \lim_{a\to\infty} \int_1^a f(x)dx,$$

when the limit exists. If $f(x) \geq 0$ and f is decreasing, prove that the limit exists if and only if $\sum_{n=1}^\infty f(n)$ converges.

Abbot exercises: 7.2.2, 7.2.7, 7.3.3, 7.4.3, 7.4.4, 7.4.10.