MAT 215: PROBLEM SET 5

DUE THURSDAY MARCH 3

Reading: Abbot, Sections 4.2-4.4.

Problem 1:

(i) Show that \( \sum_{n=1}^{\infty} n^{-p} \) converges for \( 1 < p \) and diverges for \( 0 < p \leq 1 \). **Hint:** Read Theorem 2.4.6 in Abbot.

(ii) Let \( a(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_0 \) and let \( b(x) = x^m + b_{n-1}x^{n-1} + \cdots + b_0 \) be polynomials and assume that for \( i \in \mathbb{N} \), \( b(i) \neq 0 \). Find a condition on \( m \) and \( n \) so that \( \sum_{i=1}^{\infty} \frac{a(i)}{b(i)} \) converges (prove your answer).

Abbot exercises: 2.5.5, 2.6.4, 2.7.1(a), 2.7.7.