Eden Prywes Mat 215 Spring 2022

Name: _____

Practice Midterm

- No calculators, books, or notes are permitted.
- Nothing should be on your desk but writing implements and a single one-sided reference page.
- If you have a question during the exam, you may leave the room and ask the proctor.
- You will receive points only for what is written on the numbered pages. Please use the back of pages as scratch paper.
- Please write neatly, show all your work, and justify all answers. Mysterious or illegible solutions will receive no credit.
- If you finish early, check your answers and wait until time is called.
- Please sign the Honor Pledge:

  I pledge my honor that I have not violated the Honor Code during this examination.

  _____

  No exam without a signature will be graded.

1. (5 points) State the Bolzano Weierstrass theorem.

2. (5 points) Let $E \subset \mathbb{R}$ be a set. Show that if U is an open set and $U \subset E$, then $U \subset \text{int}(E)$.

3. Let $a_n = \frac{2^n}{n!}$

   (a) (2 points) Does the sequence $(a_n)_{n\in\mathbb{N}}$ converge (prove your answer)?

   (b) (3 points) Does the series $\sum_{n=1}^{\infty} a_n$ converge (prove your answer)?

4. Let A and B be nonempty sets and suppose that for any $a \in A$ and $b \in B$, $a \leq b$.

   (a) (2 points) Show that $\sup A$ and $\inf B$ exist.

   (b) (3 points) Prove that $\sup A \leq \sup B$.

5. (a) (2 points) Give an example of sequences $(a_n)_{n\in\mathbb{N}}$ and $(b_n)_{n\in\mathbb{N}}$ so that $a_n \geq 0$, $\lim_{n\to\infty} a_n = 0$, $|b_{n+1} - b_n| \leq a_n$, but $(b_n)_{n\in\mathbb{N}}$ diverges.

   (b) (3 points) Suppose that $a_n \geq 0$ and $\sum_{n=1}^{\infty} a_n$ converges. If

$$|b_{n+1} - b_n| \leq a_n,$$

   show that $(b_n)_{n\in\mathbb{N}}$ converges to a limit.

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