FRS 157: PROBLEM SET 6

DUE WEDNESDAY, DECEMBER 7TH

Problem 1: Consider the trajectory of an object orbiting the sun, $\mathbf{r}(t)$. By using Newton's Law of Gravity and taking the dot product with velocity,

$$\ddot{\mathbf{r}} \cdot \dot{\mathbf{r}} = -GM \frac{\mathbf{r} \cdot \dot{\mathbf{r}}}{r^3}.$$

(a) Show that

$$\ddot{\mathbf{r}} \cdot \dot{\mathbf{r}} = \frac{d}{dt}\left(\frac{v^2}{2}\right),$$

where $v = \|\dot{\mathbf{r}}\|$.

(b) Show that

$$GM \frac{\mathbf{r} \cdot \dot{\mathbf{r}}}{r^3} = -GM \frac{d}{dt}\left(\frac{1}{r}\right)$$

(c) Conclude that

$$\frac{d}{dt}\left(\frac{v^2}{2} - \frac{GM}{r}\right) = 0.$$

This equation tells us that energy is conserved.

Problem 2: Apply Newton's method to Kepler's equation $t = \theta - e \sin \theta$ to predict the location of the Moon. In this case $e = 0.05$ and $a = 384,000$ km.

(a) Find the approximate location of the Moon after 1 day,

(b) after 10 days,

(c) after 13.5 days.

You may use two-dimensional coordinates where the Earth is at a focus of an ellipse and the moon starts on the x-axis at time 0.