Problem 1: Consider the trajectory of an object orbiting the sun, \( \mathbf{r}(t) \). By using Newton's Law of Gravity and taking the dot product with velocity,

\[
\mathbf{r} \cdot \dot{\mathbf{r}} = -GM \frac{\mathbf{r} \cdot \dot{\mathbf{r}}}{r^3}.
\]

(a) Show that

\[
\dot{\mathbf{r}} \cdot \dot{\mathbf{r}} = \frac{d}{dt} \left( \frac{v^2}{2} \right),
\]

where \( v = \|\dot{\mathbf{r}}\| \).

(b) Show that

\[
GM \frac{\mathbf{r} \cdot \dot{\mathbf{r}}}{r^3} = -GM \frac{d}{dt} \left( \frac{1}{r} \right)
\]

(c) Conclude that

\[
\frac{d}{dt} \left( \frac{v^2}{2} - \frac{GM}{r} \right) = 0.
\]

This equation tells us that energy is conserved.

Problem 2: Apply Newton’s method to Kepler’s equation \( t = \theta - e \sin \theta \) to predict the location of the Moon. In this case \( e = 0.05 \) and \( a = 384,000 \) km.

(a) Find the approximate location of the Moon after 1 day,

(b) after 10 days,

(c) after 13.5 days.

You may use two-dimensional coordinates where the Earth is at a focus of an ellipse and the moon starts on the \( x \)-axis at time 0.