FRS 157: PROBLEM SET 5

DUE WEDNESDAY, NOVEMBER 30TH

Reading: Newton, Principia, pages 73–89 and 103–116. Chandrasekhar, Ellipsoidal Figures of Equilibrium.

Problem 1: Let E be an ellipse with major-axis of length a and minor-axis of length b centered at its left focus.

(a) Find an equation in terms of a and e for the ellipse E. Recall that $e^2 = 1 - \frac{b^2}{a^2}$ is the eccentricity of the ellipse.

(b) Derive the equation for the ellipse in polar coordinates:

$$r = \frac{l}{1 \pm e\cos\theta},$$

where $l = a(1 - e^2)$ is the half the length of the *latus rectum*, the chord perpendicular to the x-axis that goes through a focus. **Hint:** Use the equation you found in (a), substitute $x = r\cos\theta$, $y = r\sin\theta$ and solve for r.

Problem 2: Halley's comet orbits the sun every 76 years and has an eccentricty of 0.97.

(a) Find its average distance to the sun.

(b) Find its perihelion and aphelion distances.

(c) Is Halley's comet more often closer to the sun than its average distance or farther away from the sun than its average distance? Explain your answer.

Problem 3: The distance of the moon at perigee is approximately 360000 km and at apogee is approximately 405000 km.

(a) Find the eccentricity of the Moon's orbit and the lengths of the major and minor axes for the Moon's orbit around Earth.

(b) By what percentage does the size of the moon change in the sky as it moves through its orbit? **Hint:** Draw a picture for the Moon at different distances.

(c) Compute the fastest and slowest speeds that the moon travels in the sky (in terms of angles in its orbit along the celestial sphere). **Hint:** Use Kepler's equation.