FRS 157: PROBLEM SET 4

DUE WEDNESDAY, NOVEMBER 9TH

Reading: Copernicus, Book I

Problem 1: Let an ellipse be defined as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, for a and b positive numbers. Find two circles and rates so that the sum of the motion of the two circles gives a motion of the ellipse. **Hint:** Write the motion of the ellipse as $(a \cos t, b \sin t)$ and express this as a complex number.

Problem 2: When an object is in retrograde motion on an epicycle, it appears to be stationary exactly when the sum of the directions of motion along the larger circle and smaller circles must point towards the observer (the center of the larger circle).

Suppose an object is moving along a path $c(t) = (\cos t, \sin t) + 0.25(\cos 8t, \sin 8t)$. Find the stationary points of the motion. **Hint:** The direction of motion corresponds to the velocity and is computed as the derivative.

Problem 3: Given an ellipse with major axis of length a and minor axis of length b, the chordal area between two points is defined as the area between a chord connecting two points and the ellipse.

(a) Find an expression for the chordal area between the points $(a, 0)$ and $(0, b)$.

(b) Find a general expression for the chordal area between $(a, 0)$ and another point (x, y) on the ellipse. **Hint:** Express (x, y) parameterically in terms of sines and cosines.

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