

Name: _____

Practice Midterm

- No calculators, books, or notes are permitted.
- Nothing should be on your desk but writing implements and a single one-sided reference page.
- If you have a question during the exam, you may leave the room and ask the proctor.
- You will receive points only for what is written on the numbered pages. Please use the back of pages as scratch paper.
- Please write neatly, show all your work, and justify all answers. Mysterious or illegible solutions will receive no credit.
- If you finish early, check your answers and wait until time is called.
- Please sign the Honor Pledge:

I pledge my honor that I have not violated the Honor Code during this examination.

No exam without a signature will be graded.

1. (5 points) Define Lebesgue outer measure.

2. (5 points) Show that $(\mathbb{R} \setminus \mathbb{Q}) \cap [0, 1]$ is measurable and that $m((\mathbb{R} \setminus \mathbb{Q}) \cap [0, 1]) = 1$.

3. Let \sim be a relation on \mathbb{R}^2 defined as $(x_1, x_2) \sim (y_1, y_2)$ if $x_2 = y_2$.

(a) (2 points) Show that \sim is an equivalence relation on \mathbb{R}^2 .

(b) (3 points) Find a bijective map from the set of equivalence classes of \sim to $(0, 1)$. (You must prove the map is bijective.)

4. (5 points) Prove that $f: \mathbb{R} \rightarrow \mathbb{R}$ is continuous if and only if for every closed set $C \subset \mathbb{R}$, $f^{-1}(C)$ is closed.

5. (5 points) The symmetric difference of two sets E and F is defined as $E\Delta F = E \setminus F \cup F \setminus E$. Prove that a set E is measurable if and only if for all $\epsilon > 0$, there exists an open set U so that $m^*(U\Delta E) < \epsilon$.

6. (5 points) Let $f(x) = \frac{x}{6\pi} e^{3x^2 - 2x + 1}$. Show that $f(x) = 1$ has a positive solution.

7. (5 points) Let E and F be closed sets, show that $E + F = \{x \in \mathbb{R} : x = a + b, a \in E, b \in F\}$ is measurable. **Hint:** First prove this for E and F compact.

This page left blank for more scratch work.