MAT 320: PROBLEM SET 8

DUE MONDAY NOVEMBER 15

Problem 1: Find the integral of the Cantor-Lebesgue function defined in class over the interval $[0,1]$.

Problem 2: Let $f\colon [0,1] \to \mathbb{R}$ be measurable. Show that for all $\epsilon > 0$, there exists a measurable set $A \subset [0,1]$ such that $m([0,1] \setminus A) < \epsilon$ and f is bounded on A.

Problem 3: Let $\phi(x) = ax + b$, show that for any measurable function $f\colon \mathbb{R} \to \mathbb{R}$,

$$\int_{\mathbb{R}} f(x) = |a| \int_{\mathbb{R}} f(\phi(x)).$$

Problem 4: Let $f\colon \mathbb{R} \to \mathbb{R}$ and $g\colon \mathbb{R} \to \mathbb{R}$ be measurable functions. Suppose that f is absolutely integrable and g is bounded almost everywhere. Define the convolution between f and g as

$$f * g(x) = \int_{\mathbb{R}} f(x-y)g(y)dy.$$

(i) Show that the convolution converges for all $x \in \mathbb{R}$.

(ii) Show that $f * g(x) = g * f(x)$.

(iii) Prove that $f * g$ is a continuous function on $\mathbb{R}$. **Hint:** First consider f defined on a bounded interval and use problem 3.

Problem 5: Chapter 4.4 Problem 33.

Problem 6: Chapter 4.5 Problem 38.