

## MAT 320: PROBLEM SET 7

DUE MONDAY NOVEMBER 8

**Problem 1:** Let  $f: [0, 1] \rightarrow [0, \infty)$  be a Riemann integrable function. In other words,

$$\int_0^1 f(x)dx = \inf_{0=t_0 < \dots < t_n=1} \sum_{i=1}^n \sup_{x \in [t_{i-1}, t_i]} f(x) |t_i - t_{i-1}| = \sup_{0=t_0 < \dots < t_n=1} \sum_{i=1}^n \inf_{x \in [t_{i-1}, t_i]} f(x) |t_i - t_{i-1}|.$$

Show that  $f$  is measurable, Lebesgue integrable and that the Lebesgue integral equals the Riemann integral.

**Problem 2:** Let  $f: E \rightarrow [0, \infty]$  be absolutely integrable. Show that the following sum converges:

$$\sum_{k=0}^{\infty} km(\{x \in E : k < f(x) \leq k + 1\}).$$

**Problem 3:** Chapter 3.1 Problem 8.

**Problem 4:** Chapter 3.2 Problem 14.

**Problem 5:** Chapter 3.2 Problem 15.

**Problem 6:** Chapter 4.3 Problem 17.