MAT 320: PROBLEM SET 6

DUE MONDAY NOVEMBER 1

Problem 1: Let $\{f_n\}_{n\in\mathbb{N}}$ be a sequence of measurable functions defined on $E \subset \mathbb{R}$. Show the following functions are measurable:

(i) $\sup_{n\in\mathbb{N}} f_n(x)$

(ii) $\inf_{n\in\mathbb{N}} f_n(x)$

(iii) $\limsup_{n\to\infty} f_n(x)$

(iv) $\liminf_{n\to\infty} f_n(x)$

Hint: You may use the fact that the pointwise limit of measurable functions is measurable.

Problem 2:

(i) If $f\colon \mathbb{R} \to \mathbb{R}$ is measurable and $g\colon \mathbb{R} \to \mathbb{R}$ is continuous, is $f \circ g$ measurable?

(ii) Let $f_1, \ldots, f_n$ be measurable functions. Show that $\min\{f_1, \ldots, f_n\}$ and $\max\{f_1, \ldots, f_n\}$ are measurable.

Problem 3: Chapter 2.5 Problem 28.

Problem 4: Chapter 2.7 Problems 38 and 39.

Problem 5: Chapter 3.1 Problem 1.