MAT 320: PROBLEM SET 5

DUE MONDAY OCTOBER 11

Problem 1: Let Σ be a collection of subsets of $\mathbb{R}$. Σ is a σ-Algebra if (a) $\emptyset \in \Sigma$, (b) if $A \in \Sigma$, then $\mathbb{R} \setminus A \in \Sigma$ and (c) if $\{A_k\}_{k\in\mathbb{N}}$ are subsets of $\mathbb{R}$ so that for each $k \in \mathbb{N}$, $A_k \in \Sigma$, then $\cup_{k\in\mathbb{N}} A_k \in \Sigma$.

(i) Prove that if $\{A_k\}_{k\in\mathbb{N}}$ are subsets of $\mathbb{R}$ so that for each $k \in \mathbb{N}$, $A_k \in \Sigma$, then $\cap_{k\in\mathbb{N}} A_k \in \Sigma$.

(ii) The σ-algebra generated by a collection of sets $\mathcal{A}$, is the smallest σ-algebra that contains $\mathcal{A}$. What is the σ-algebra generated by $A = \{\emptyset\}$?

(iii) The Borel σ-algebra, $\mathcal{B}$, is defined as the σ-algebra generated by the open sets of $\mathbb{R}$. Show that if $f\colon \mathbb{R} \to \mathbb{R}$ is continuous and $A \in \mathcal{B}$, then $f^{-1}(A) \in \mathcal{B}$.

Problem 2: Let $E \subset \mathbb{R}$, show that if for all $\epsilon > 0$ there exists a closed set $F \subset E$ so that $m^*(E \setminus F) < \epsilon$, then E is measurable.

Problem 3: Chapter 2.3 Problem 14.

Problem 4: Chapter 2.3 Problem 15.

Problem 5: Chapter 2.4 Problem 17.

Problem 6: Chapter 2.4 Problem 18. The definitions for G_δ and F_σ sets can be founded on page 39 of the text.