MAT 320: PROBLEM SET 4

DUE MONDAY OCTOBER 4

Problem 1: Define the distance between two sets $E, F \subset \mathbb{R}$ as

$$d(E, F) = \inf\{|x - y| : x \in E \text{ and } y \in F\}.$$

Show that if E is compact, F is closed and $E \cap F = \emptyset$, then $d(E, F) > 0$.

Problem 2: Let $E \subset [0, 1]$ be a set with $m^*(E) = 0$ and let $f : [0, 1] \to \mathbb{R}$ be a Lipschitz function. Show that $m^*(f(E)) = 0$.

Problem 3: Give an example of a set E that satisfies the following property:

$$m^*(E) \neq \sup\{m^*(U) : U \subset E, U \text{ is open}\}.$$

Problem 4: Chapter 1.6, Problem 50.

Problem 5: Chapter 1.6, Problem 59.

Problem 6: Chapter 2.2 Problem 10.