

## MAT 320: PROBLEM SET 10

DUE FRIDAY DECEMBER 3

**Problem 1:** Let  $f: [0, 1] \rightarrow \mathbb{R}$  be continuous and of bounded variation. Show that  $f$  is the difference of two monotonic increasing **continuous** functions  $g, h: [0, 1] \rightarrow \mathbb{R}$ .

**Problem 2:** Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be an integrable function. Show that  $F(x) = \int_{(-\infty, x]} f$  is absolutely continuous (and hence of bounded variation) and that  $TV(F) = \int_{\mathbb{R}} |f|$ .

**Problem 3:** Prove that  $x^p$  is absolutely continuous on  $[0, 1]$  for  $p > 0$ .

**Problem 4:** Chapter 6.2 Problem 16.

**Problem 5:** Chapter 6.2 Problem 23.

**Problem 6:** Chapter 6.3 Problem 35.

**Problem 7:** Chapter 6.4 Problem 38.

**Problem 8:** Chapter 6.4 Problem 41.