Problem 1: Show that the composition of two injective maps is injective, the composition of two surjective maps is surjective and the composition of two bijective maps is bijective.

If \( g: A \rightarrow B \) is surjective and \( f: B \rightarrow C \) is injective, is \( f \circ g \) injective or surjective? What about if \( f \) is surjective and \( g \) is injective?

Problem 2: Let \( f: A \rightarrow B \) be a map. Show that the following relation on \( A \) is an equivalence relation. If \( x, y \in A \), then \( x \sim y \) if \( f(x) = f(y) \).

Conversely, show that every equivalence relation can arise from such a map. That is, if \( \sim \) is an equivalence relation on \( A \), then there exists a set \( B \) and a map \( f: A \rightarrow B \) so that \( x \sim y \) if and only if \( f(x) = f(y) \).

Hint: Use the equivalence classes on \( A \).

Problem 3: Show that \( \mathbb{Q} \) as defined in class satisfies the field axioms. Show that \( \mathbb{Z} \) as defined in class does not satisfy the field axioms.

Problem 4: Show that between any two points in \( \mathbb{R} \) there exists a rational number and an irrational number.

Hint: You may use without proof that \( \sqrt{2} \) is irrational.

Problem 5: Chapter 1.1 Question 3.

Problem 6: Chapter 1.2 Question 14.