MAT 320: PROBLEM SET 1

DUE MONDAY SEPTEMBER 13

Problem 1: Show that the composition of two injective maps is injective, the composition of two surjective maps is surjective and the composition of two bijective maps is bijective.

If $g\colon A \to B$ is surjective and $f\colon B \to C$ is injective, is $f \circ g$ injective or surjective? What about if f is surjective and g is injective?

Problem 2: Let $f\colon A \to B$ be a map. Show that the following relation on A is an equivalence relation. If $x, y \in A$, then $x \sim y$ if $f(x) = f(y)$.

Conversely, show that every equivalence relation can arise from such a map. That is, if $\sim$ is an equivalence relation on A, then there exists a set B and a map $f\colon A \to B$ so that $x \sim y$ if and only if $f(x) = f(y)$.

Hint: Use the equivalence classes on A.

Problem 3: Show that $\mathbb{Q}$ as defined in class satisfies the field axioms. Show that $\mathbb{Z}$ as defined in class does not satisfy the field axioms.

Problem 4: Show that between any two points in $\mathbb{R}$ there exists a rational number and an irrational number.

Hint: You may use without proof that $\sqrt{2}$ is irrational.

Problem 5: Chapter 1.1 Question 3.

Problem 6: Chapter 1.2 Question 14.