

HW4

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$$f(z) = \frac{z+2}{z}$$

(a) $C_1 = \{z = ze^{i\theta}, 0 \leq \theta \leq \pi\}$

$$\int_{C_1} f(z) dz = \int_0^{\pi} f(ze^{i\theta}) ze^{i\theta} i d\theta = \int_0^{\pi} \frac{ze^{i\theta} + 2}{ze^{i\theta}} i ze^{i\theta} d\theta =$$

$$= 2i \int_0^{\pi} (e^{i\theta} + 1) d\theta = 2i \left[\frac{e^{i\theta}}{i} \Big|_0^{\pi} + \pi \right] =$$

$$= 2\pi i + 2 \left(\frac{e^{i\pi}}{i} - 1 \right) = -4 + 2\pi i$$

(b) $C_2 = \{z = ze^{i\theta}, \pi \leq \theta \leq 2\pi\}$

$$\int_{C_2} f(z) dz = \dots = 2i \left[\frac{e^{i\theta}}{i} \Big|_{\pi}^{2\pi} + \pi \right] = 2\pi i + 2 [1 - (-1)] = 4 + 2\pi i$$

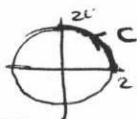
(c) $C_3 = \{z = ze^{i\theta}, 0 \leq \theta \leq 2\pi\}$

$$\int_{C_3} f dz = \int_{C_1} + \int_{C_2} = 4\pi i$$

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$$\int_C \frac{dz}{z^2-1}$$

$C = \{ \text{arc of } |z|=2 \text{ from } z=2, \text{ to } z=2i, \text{ that lies in the 1st quadrant} \}$



observe: length(C) = $2 \cdot \frac{\pi}{2} = \pi$

$$\frac{1}{|z^2-1|} \text{ when } |z|=2: \frac{1}{|z^2-1|} \leq \frac{1}{|z|^2-1} = \frac{1}{4-1} = \frac{1}{3}$$

$$\Rightarrow \left| \int_C \frac{dz}{z^2-1} \right| \leq \int_C \frac{1}{|z^2-1|} |dz| \leq \frac{1}{3} \cdot \pi = \frac{\pi}{3} \quad \square$$