Homework assignment #7

Math 317

Due Friday, April 1

1. Solve the following problems from the textbook: P 153 ex. 2, P 162 ex. 1(a-c), 2, 3, 5 (note: z_0 can be both inside and outside C), 7
2. Show, using Cauchy's theorem and its variants found in §46 that $g(z) = \frac{1}{z(z-1)}$ has an antiderivative in $\mathbb{C} \setminus \{z \text{ real}, 0 \le z \le 1\}$ (cf. with Assignment #6, ex. 2d where the antiderivative is given explicitly).
3. If you want, you may submit a solution to ex. 2d from Assignment #6; if this is better than your original solution this will count instead of your original solution to 2d.
4. Suppose that $g(z)$ is analytic everywhere in $\mathbb{C}$ except possibly at the finitely many points $z_1,\ldots,z_n$. Let $\rho = \min_{i \neq j} |z_i - z_j|$. Suppose that C_i for $i = 1, \ldots, n$ is the positively oriented simple closed contour along $|z - z_i| = \rho/3$. Show that if

$$\int_{C_i} g(z)dz = 0 \qquad \text{for } i = 1, \ldots, n$$

then g has an antiderivative in $\mathbb{C} \setminus \{z_1, \ldots, z_n\}$.