Homework assignment #6

Math 317

Due Friday, March 25

Part I: Solve All of the following [90 points/100]:

1. Solve the following problems from the textbook: P 141 ex. 2, P 153 ex. 1, P 156 ex. 7,

2. Let \( g(z) = \frac{1}{z(z-1)} \).
   
   (a) show \( g \) is analytic in \( \mathbb{C} \setminus \{0, 1\} \).
   
   (b) Let \( C \) be the positively oriented closed contour along the circle \( |z| = 1/2 \). Calculate \( \int_C g(z)dz \). \textbf{Hint:} use the formula
   \[
   \frac{1}{z(z-1)} = \frac{1}{z-1} - \frac{1}{z} \quad \text{(0.1)}
   \]

   (c) deduce that \( g \) does not have an antiderivative in \( \mathbb{C} \setminus \{0, 1\} \).
   
   (d) find an antiderivative for \( g \) in the domain \( D = \mathbb{C} \setminus \{ x : x \text{ real, } 0 \leq x \leq 1 \} \).
   \textbf{Hint:} first find, using formula (0.1) an antiderivative for \( g \) in the smaller domain \( \mathbb{C} \setminus \{ x : x \text{ real, } -\infty < x \leq 1 \} \).

3. A set \( D \subset \mathbb{C} \) is said to be \textbf{convex} if for every \( z, w \in D \), all points of the line segment connecting \( z \) and \( w \) are in \( D \).

   Prove that if \( D \) is a convex domain (i.e. a domain that is also convex), \( f \) a continuous function on \( D \), and if \( \int_C f(z)dz = 0 \) for every \textbf{triangular} contour completely in \( D \) then \( f \) has an antiderivative in \( D \).

   \textbf{Hint:} in class we proved that if \( \int_C f(z)dz = 0 \) over every closed contour then \( f \) has an antiderivative in \( D \) (our proof is essentially identical to the one in the textbook in section 42). If you define \( F(z) \), the candidate for an antiderivative of \( f \) correctly, the proof of this exercise is very similar.
Part II: Solve AT LEAST ONE of the following questions. Solve more for a BONUS.

1. [10 points] Let $D = \{ z : 100 < |z| < 101 \}$. Find a function $f$ so that $\int_C f(z)dz = 0$ for every triangular contour completely in $D$ but $f$ does not have an antiderivative in $D$ (don’t forget to prove your function works!). Why doesn’t this contradict (3) of Part I?

2. [25 points] complete the proof of Goursot’s theorem for a rectangle given in class. Write it down like we did it in class, where we reduced everything to a question whether a certain algorithm terminates in a finite number of steps. You are welcome to look at the textbook (or at any other source), as long as your answer follows the outline I gave in class.

Please write down a complete proof.


ENJOY YOUR BREAK!