Homework assignment #6

Math 317

Due Friday, March 25

Part I: Solve All of the following [90 points/100]:

1. Solve the following problems from the textbook: P 141 ex. 2, P 153 ex. 1, P 156 ex. 7,

2. Let $g(z) = \frac{1}{z(z-1)}$.

   (a) show g is analytic in $\mathbb{C} \setminus \{0, 1\}$.

   (b) Let C be the positively oriented closed contour along the circle $|z| = 1/2$. Calculate $\int_C g(z)dz$. *Hint:* use the formula

   $$\frac{1}{z(z-1)} = \frac{1}{z-1} - \frac{1}{z} \tag{0.1}$$

   (c) deduce that g does not have an antiderivative in $\mathbb{C} \setminus \{0, 1\}$.

   (d) find an antiderivative for g in the domain $D = \mathbb{C} \setminus \{x : x \text{ real}, 0 \leq x \leq 1\}$. *Hint:* first find, using formula (0.1) an antiderivative for g in the smaller domain $\mathbb{C} \setminus \{x : x \text{ real}, -\infty < x \leq 1\}$.

3. a set $D \subset \mathbb{C}$ is said to be **convex** if for every $z, w \in D$, all points of the line segment connecting z and w are in D.

   Prove that if D is a convex domain (i.e. a domain that is also convex), f a continuous function on D, and if $\int_C f(z)dz = 0$ for every **triangular** contour completely in D then f has an antiderivative in D.

   Hint: in class we proved that if $\int_C f(z)dz = 0$ over every closed contour then f has an antiderivative in D (our proof is essentially identical to the one in the textbook in section 42). If you define $F(z)$, the candidate for an antiderivative of f correctly, the proof of this exercise is very similar.

Part II: Solve AT LEAST ONE of the following questions. Solve more for a BONUS.

1. [10 points] Let $D = \{z : 100 < |z| < 101\}$. Find a function f so that $\int_C f(z)dz = 0$ for every triangular contour completely in D but f does not have an antiderivative in D (don't forget to prove your function works!). Why doesn't this contradict (3) of Part I?

2. [25 points] complete the proof of Goursot's theorem for a rectangle given in class. **Write it down like we did it in class**, where we reduced everything to a question whether a certain algorithm terminates in a finte number of steps. You are welcome to look at the textbook (or at any other source), as long as your answer follows the outline I gave in class.

   Please write down a complete proof.

3. [25 points] P. 154, ex 4.

ENJOY YOUR BREAK!