Homework assignment #5

Math 317

Due Friday, March 11

Solve the following problems from the textbook:

1. P. 94 ex. 6, P. 96 ex. 2
2. P. 116 ex. 7. Show in addition that P_n is a polynomial of degree n in x.
3. P. 121 ex. 5, P. 130 ex. 10,P. 134 ex. 6,7 (in 7(a) prove the two stated inequalities), P. 142 ex. 3

Additional questions:

1. Let C be the closed positively oriented contour along $|z| = 1$. Prove that for any function $g(z)$,
$$\overline{\int_C g(z)dz} = \int_C \overline{g(z)} z^{-2} dz.$$
2. Let C be a contour in $\mathbb{C} \setminus \{0\}$, given by the parametrization $z(s)$, $0 \le s \le T$. For every $0 \le t \le T$ consider the contour C_t given by $z(s)$, $0 \le s \le t$. We define $I(t) = \int_{C_t} \frac{dz}{z}$.
   (a) by using P. 121, ex. 5 and the fundamentals theorem of the calculus, show that the function $g(t) = \exp(I(t))/z(t)$ satisfies $g'(t) = 0$, i.e. is a constant.
   (b) conclude that for any contour C between a point z_0 and a point $z_1 \in \mathbb{C}$ that avoids 0
   $$\exp \int_C \frac{dz}{z} = \frac{z_1}{z_0}.$$

(c) deduce that if C is a closed contour avoiding 0,

$$\int_C \frac{dz}{z} = 2\pi i n \tag{0.1}$$

for some integer n. This integer is called the **index** or **winding number** of C around zero.

(d) (BONUS) try proving (0.1) using antiderivatives. You may need to restrict the type of contours you are considering.