Homework assignment #10

Math 317

Due Friday, April 22

1. Solve the following problems from the textbook: P 230 ex. 2, 4, P 238 ex. 1, 3, 5, P 245 ex. 4, 5, 7

2. Let g be a real valued function of a real variable, Riemann integrable (if you want — continuous), with $\int_{-\infty}^{\infty} |g(t)|\, dt < \infty$. Recall that the Fourier transform $\hat{g}$ of g is defined as the function

$$\hat{g}(s) = \int_{-\infty}^{\infty} \exp(-ist)g(t)dt \qquad \text{for every } s \in \mathbb{R}. \tag{0.1}$$

Suppose now that $g(t) = 0$ for all t outside of some finite interval $[a, b] \subset \mathbb{R}$. Such functions are called **compactly supported**. Prove that the function $\hat{g}(s)$ (a complex valued function of a real variable) cannot be compactly supported.

Directions: if $g(t)$ is compactly supported, (0.1) actually makes sense for general complex s, not just for those $s \in \mathbb{R}$ where it was originally defined. To avoid confusion, we let $h(s) = \int_{-\infty}^{\infty} \exp(-ist)g(t)dt$ be the complex valued function of a complex variable defined in this way, so $h(s) = \hat{g}(s)$ for $s \in \mathbb{R}$.

Similarly to the way we showed in class using Cauchy's integral formula for f', (where f is an analytic function), that f' was itself analytic it is quite easy to show that $h(s)$ is entire. (You may assume this is true without proving it)

Now use the theorems we have discussed in class regarding zeros of analytic functions to get a contradiction to $\hat{g}(s)$ having compact support.