

Homework assignment #1

due

2-11-2005

From textbook:

P. 11 ex. 3, 4

P. 13-14 ex. 7, 10

P. 21 ex. 1, 5, 10

also: review section 10

additional questions:

1. We defined the closure of a subset $S \subset \mathbb{C}$ to be

$$cl(S) = S \cup \{\text{boundary pts of } S\}.$$

Prove that (i) this set is closed.

(ii) any closed set A containing S contains also $cl(S)$.

2. (a) show that for any z

$$|z| = \max_{-\pi < \theta \leq \pi} \operatorname{Re}(z e^{i\theta})$$

(b) use (a) to give another proof of the triangle inequality

$$|z_1 + z_2| \leq |z_1| + |z_2|$$

3. a complex number of the form $n+im$, $n \in \mathbb{Z}$, $m \in \mathbb{Z}$ is called a Gaussian integer.

1. show that the set of Gaussian integers is closed under $\cdot, +$.
for which Gaussian integer w is w^{-1} also a Gaussian integer.

2. we say that " n is the sum of two squares" if

$$n = a^2 + b^2 \quad a, b \in \mathbb{Z}.$$

show that if n & m are sums of two squares, so is nm (hint: what is $|w|^2$ for w a Gaussian integer?)