Math. 522: Homework 3, due October 22

1. Let $\Omega \subset \mathbb{R}^n$ be a bounded domain. If u is a measurable function so that $|u|^p \in L^1(\Omega)$ for some $p \in \mathbb{R}$, define

$$\Phi_p(u) = \left[\frac{1}{|\Omega|}\int_\Omega |u|^p dx\right]^{\frac{1}{p}}.$$

Show that

$$(1) \quad \lim_{p\to\infty} \Phi_p(u) = \sup_\Omega |u|$$

$$(2) \quad \lim_{p\to-\infty} \Phi_p(u) = \inf_\Omega |u|$$

$$(3) \quad \lim_{p\to 0} \Phi_p(u) = \exp\left[\frac{1}{|\Omega|}\int_\Omega \log|u| dx\right]$$

2. Problem 17 from Chapter 5 of Evans. It deals with showing that $|u|, u^+, u^-$ are in $W^{1,p}(\Omega)$ if $u \in W^{1,p}(\Omega)$, $1 \le p \le \infty$, Ω bounded. It requires to compute derivatives as well.

3. Problem 18 from Chapter 5 in Evans. It requires to prove the continuous embedding $L^\infty(\mathbb{R}^n) \subset H^s(\mathbb{R}^n)$ for $s > \frac{n}{2}$ using the Fourier transform.

4. Problem 2 from Chapter 6 in Evans. This asks you to prove the existence of weak solutions to the Dirichlet problem for the biharmonic equation.

5. Problem 3 from Chapter 6 in Evans. It is about the Neumann problem for the Laplacian.

6. Problem 8 from Chapter 6 in Evans. It is about convex functions of bounded weak solutions of homogeneous elliptic equations in divergence form being subsolutions.