Session 2: Square Roots and the Pythagorean Theorem

1. Tosafot, Sukkah 8a

   Whatever is one cubit square has a diagonal of 1 2/5 cubits. This figure is not exact, for it’s a little more. If you make a square of ten by ten, and divide it in two and again, you will find in it four squares of five by five. Now divide each of those squares with the diagonals that connect midpoints of the sides of the large square. You will find in the inside square an area of fifty square cubits, since it’s half of the outer one, since the 5 by 5 squares were split along their diagonals. But if the figure were only 1 2/5 cubits, it would be a 7 by 7 square, and this isn’t half the area of the outer square. It’s only 49 square cubits!

2. Babylonian clay tablet YBC 7289 (c. 1800–1600 BCE)

   \[1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} = 1.41421296...\]
There are three types of triangles: right, acute, and obtuse. What is a right triangle? Its two short sides are multiplied into themselves [and added together. The long side, which is the base, is multiplied into itself. The last square] is equal to the first.

4. Mishnah Eiruvin, Chapter 2

*Mishnah 3.* R. Judah said: [One may carry objects in an enclosure] as large as two *beit se’ah,* but they [the Sages] said to him: [the limit of] two *beit se’ah* was prescribed for a garden or a storage area only; but if [the enclosure] was a cattlepen, a sheepfold, a backyard, or a courtyard, it may be [as big as] five or ten *beit kor*…

*Mishnah 5.* R. Judah b. Baba said further: it is permitted to move objects in a garden or a storage area that is seventy cubits and a fraction by seventy cubits and a fraction and is surrounded by a wall ten handbreadths high, provided it contains a watchman’s hut or a dwelling place or it is adjacent to a town. R. Judah said: even if it contains only a cistern, a ditch, or a cave it is permitted to move objects within it. R. Akiba said: even if it contains none of these it is permitted to move objects within it, provided its area is no more than seventy cubits and a fraction by seventy cubits and a fraction. R. Eliezer said: if its length exceeds its breadth even by a single cubit it is not permitted to move any objects within it. R. Yose said: even if its length is twice its breadth it is permitted to move objects within it.
5. Babylonian Talmud, Eiruvin 23b

It seems that R. Akiva agrees with the Sages! – There’s a small difference between them, as R. Judah taught: There’s a small difference [between two bet se’ah and a square of] 70 and a fraction, but the Sages didn’t state it precisely. And how much is two bet se’ah? – Like the courtyard of the Temple. How do we know this? – R. Judah said: As it says in Scripture (Exodus 27:18), “The length of the courtyard was 100, and the width was 50, in 50.” The Torah is saying, take away 50 and surround them with 50. But what is the plain meaning? Abaye said: The Temple stood at the edge of 50, so that there were 50 cubits in front of it, and 20 in the remaining directions.

6. Jerusalem Talmud, Eiruvin 20b

R. Shmuel bar Nachman said in the name of R. Yonatan: From the dimensions of the Temple we learn: “The length of the courtyard was 100 cubits, and the width was 50 in/by 50.” And 50 times 100 is 5000. 70 by 70 is 5000 less 100, so we learn: 70 and a fraction. Shmuel taught: 70 2/3 cubits is two small ones. 70 times 2/3 and 70 times 2/3 are 140/3 and 140/3, which is 93 1/3. Coming out are 4/9 on the four corners. What remains is 19/3 less 1/9, as taught. This is a close approximation; the sages could not determine the exact dimensions of a [square] storage area whose area is two bet se’ah.
Therefore, also add one-third to the lot; thus the area which was originally 3750 becomes 4789 square cubits. The claim is that make the fraction 5/7 like I said, and multiply 70 5/7 by 50 5/7, you will get approximately 5000 ½. But if you make the fraction 2/3 like in the Yerushalmi, the area will be 4993 7/9. This is the difference between R. Yehudah ben Bava, who said “70 and a fraction by 70 and a fraction,” and R. Akiva, who said “only 70 and a fraction by 70 and a fraction,” in addition to the presence of a hut or a booth. R. Yehuda ben Bava is more precise in computation, so that it should be two full beit se’ah, so he makes the fraction 5/7 like we said, or something more precise than that. R. Akiva gives a more approximate figure, and makes the fraction 2/3 or something close to that, so that the area is close to two beit se’ah, and thus is seen in their words elsewhere.

7. Rambam, Commentary on the Mishnah, Eiruvin 2:5

We already explained that the Temple courtyard is two beit se’ah. And you know that the area of the Temple courtyard is 5000 square cubits, since its length is 100 and its width is 50, and any place whose area is 5000 square cubits is two beit se’ah, whether it’s a circle or a square or a triangle or any other shape. But it is not possible to know the side a square with right angles whose area is 5000 square cubits exactly, since 5000 is a number that doesn’t have a [rational] square root. Its root is approximately 70 5/7, and the significance of this number is like I explained above about the relation of the diameter of a circle to its circumference … and this isn’t because of a lack of our knowledge, but the nature of this number. And therefore they said “70 and a fraction” (shirayim). If you make the fraction 5/7 like I said, and multiply 70 5/7 by 50 5/7, you will get approximately 5000 ½. But if you make the fraction 2/3 like in the Yerushalmi, the area will be 4993 7/9. This is the difference between R. Yehudah ben Bava, who said “70 and a fraction by 70 and a fraction,” and R. Akiva, who said “only 70 and a fraction by 70 and a fraction,” in addition to the presence of a hut or a booth. R. Yehuda ben Bava is very precise in computation, so that it should be two full beit se’ah, so he makes the fraction 5/7 like we said, or something more precise than that. R. Akiva gives a more approximate figure, and makes the fraction 2/3 or something close to that, so that the area is close to two beit se’ah, and thus is seen in their words elsewhere.

8. R. Saadia Gaon (882-942), Kitāb al mawārīth (Laws of inheritance) (trans. Solmon Gandz, 1943)

[Background: Finding the diameter of a circle whose area is 9 kab = 1.5 se’ah = 3500 square cubits. The claim is that \(\sqrt{5000} \approx 70\frac{4}{60}\).]

If it is circular, we must add one-third to the measure of these 9 Kab, and therefore, also add one-third to the lot; thus the area which was originally 3750 becomes 5000. So that if you seek the measure of (the diameter of) the circular field, you will
multiply \(70 + \frac{2}{3} + \frac{1}{2} \times \frac{1}{20}\) by itself and find the true sum. Its explanation is as follows: \(7 \times 7 = 49\); \(70 \times 70 = 4900\). There still remains \(\frac{2}{3} + \frac{1}{2} \times \frac{1}{10}\): \(\frac{2}{3} \times 70 = \frac{140}{3} = 46 \frac{2}{3}\); \(\frac{1}{2} \times \frac{1}{10} \times 70 = \frac{7}{2} = 3 \frac{1}{2}\); together they are \(50 \frac{1}{6}\); double it, for it is multiplied by itself, and you get \(100 \frac{1}{3}\). Throw away the one-third to the other fractions and \(100\) remains; add it to the above and you get \(5000\) conforming to the computation. Such is the procedure for the circular field.

9. Babylonian Talmud, Eiruvin 56b

[Background: The Sabbath limit (\textit{t'chum Shabbat}) of a town extends 2000 cubits beyond the town boundary. If the town is circular, we first circumscribe a square around it, parallel to the cardinal directions; the \textit{t'chum} is a square extending 2000 cubits beyond these fictitious boundaries.]

When one squares a town, one makes in the shape of a square tablet, and then makes the \textit{t'chum} in the shape of a square tablet. In measuring, one should not measure from the corner point [of the fictitious town boundaries], since then one loses the corners, but rather attaches a square 2000 cubits by 2000 cubits diagonally in each corner. Thus, the town gains 400 cubits in each corner, the \textit{t'chum} gains 800 cubits in each corner, and the town and the \textit{t'chum} together gain 1200 cubits in each corner. Abaye says: This applies in a town 2000 by 2000 cubits.

10. Mishnah Kilayim 5:5

If one plants or maintains vegetables [in a vineyard], this ruins [lit. sanctifies] 45 vines. When does this apply? When they are planted 4 by 4 or 5 by 5. If they were planted 6 by 6 or 7 by 7, it ruins 16 cubits in all directions, in a circle rather than a square.
forbidden. For if one plants 7 rows of vines, with 7 vines in each row, spaced 4 cubits
apart, one cannot know how many "chametz" is planted, since the edges of the vines
are indistinguishable from a single row of chametz.

However, the Scripture (Deuteronomy 18:14) states: "If you count..."
This is because the edges of the vines are indistinguishable from a single row of chametz.

Ramban explains that because the edges of the vines are indistinguishable from a single row of chametz,
the edges of the vines are not considered chametz.

Therefore, the edges of the vines are not considered chametz.

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apart, you will have 49 vines in a square 24 by 24 cubits, since there are 6 gaps of 4 between each row. And if one plants [vegetables] around the middle vine of the fourth row — i.e., the middle vine of all the vines — it ruins 16 cubits in all directions, as taught in the second half, i.e. a circle 32 cubits in diameter. All 49 vines are inside this circle except the four corner ones, for “a square of a cubit is a cubit and two fifths in diagonal.” We find that a square of 24 has a diagonal that is 9.6 cubits greater [i.e. 33.6 cubits], so the corners are outside the 32.

And likewise when the vines are spaced 5 cubits apart, it ruins 45 vines. For the length of the row, with 6 spaces, is 30 cubits. Apart from the corners, not one of the vines is more than 16 cubits from the middle. For each corner is 5 apart, and you find that from the center vine to the farthest vine is only half the diagonal of 30 by 20. And it is easy to make a picture and to understand. The diagonal of 30 by 20 does not exceed the [width of the] rectangle more than the diagonal of 20 by 20 exceeds. Know that if you tie a thread on the corner and bring it on the diagonal to the end of 20, and then straight down 10 cubits, we find the length of the thread is 38 cubits, for the diagonal of 20 by 20 exceeds the width by 8. And if you stretch it diagonally from one corner to the other, it is totally clear that it shortens greatly. . . . And from our Mishna, we can conclude that the diagonal of 30 by 20 does not exceed the width by more than 2 cubits [i.e. isn’t greater than 32], for the distance [from the vine next to the corner] to the middle vine is 16 cubits.

People who are wise in geometry have said that the squares of the two sides are the square of the diagonal. That is, you measure the width and make a square of that size, and you measure the length and make a square of that size, and you measure the diagonal and make a square of that size. Then the area of the diagonal equals the area of the other two squares. To see this, make a square of 100 by 100, and divide it into four squares of 50 by 50, and then divide each of them diagonally. Your eyes will see that the inner square has half the area [of the outer square], for we’ve divided each of the 50 by 50 squares in half. Thus, the squares of the two sides are twice 50 by 50. And this also the square of the diagonal, as I showed you, since the inner square is half of the outer one, or 100 by 50, which is twice 50 by 50.

I’ll show you another way also. Two squares of 50 by 50 can be split on their diagonals, and then you put these four triangles together with their diagonals on the outside. You’ve made a square out of these two squares of 50 by 50, divided along its diagonals. But it is a problem, for the sages said in Eiruvin (23b) that the Temple courtyard was 100 by 50, which is the same area as 70 and a fraction. According to this computation, this square is only 70 by 70 reduced, for a one cubit square is one and two fifths cubits in diagonal, so the diagonal of 50 by 50 is only 70. But this depends on something that doesn’t add up exactly: the statement that a one cubit square is one and two fifths cubits in diagonal. As you can see, with these two squares, one inside the other, using the figure of one and two fifths, the inner one is 70 by 70. Its diagonal is 100 since it’s the width of the outer square, but it’s only 98 by computation. The sages were not concerned with such small amounts, so both the words of the sages and the words of the geometers are valid.

And even though I’ve proven this for a square whose sides are equal, it can’t be proved for a rectangle whose length is greater than its width, and therefore, there is nothing to it. For the diagonal of 30 by 20 is less than 32, as our Mishnah shows. And if
you make a square of 32 by 32, it’s too small; it’s not the size of two squares, one 30 by 30 and one 20 by 20.

Rambam explained that it ruins the vines next to the corner vines even though they are more than 16 cubits away from the middle vine, because we view the 16 cubits around the middle vine as if they were filled with vegetables, and this circle is 32 in diameter and sticks out of the 7 rows of 5 cubits by one cubit on each side, since there are 6 gaps between 7 rows, each 5 wide, and thus it’s a 30 by 30 square. The diagonal is 32 cubits, so the circle sticks out of the 7 rows by one cubit, and thus comes up to the tending region of the eighth and ninth rows. Therefore, we must add to the circle 4 cubits in every direction, so it’s a circle of 40, since we view the circle of 32 as if it were filled with vegetables everywhere. Wherever it enters the tending region, it ruins.
forbid the 8 vines next to the corners, even though they are more then 16 cubits from the middle vine.

13. Rambam, Mishnah Kilayim 5:5

We have now explained the details of this law with the help of God. Contemplate them carefully, for when a great rabbi of the rabbis of the Talmud was asked about this, his answer was confusing, and there were many questions left unresolved. But then Heaven helped, and we were ordered to explain them to you.


I have found a text in which the Rosh asked R. Yisrael b. Yosef, who was very knowledgable (chacham b’chochmot), who is correct in this matter (Rambam or R. Shimshon). R. Yisrael writes: “My lord, my master, may your greatness be like your talent and ability. I consulted Rambam’s (z”)l commentary on the Mishnah, and I saw that that is knowledge is extraordinary. His explanation provides excellent guidance, but it seems, because of my lack of knowledge, that he elaborates too much and the language is not clear and would be confusing to someone who doesn’t study it carefully. But it all makes perfect sense. One may not stray left or right from his approach. Thus, I will explain the entire argument and not add anything, save for simplifying the language…

“This is what I have understood from the Rambam’s (z”)l explanation of this Mishna, and it is Torah l’Moshe b’Sinai. But let not my lord be seduced by R. Shimshon’s (z”)l explanation of this Mishnah, and do not pay attention to it at all, for it is beyond his honor, a disgrace founded on a vacuous line [of thought] and a nonsensical
foundation. What the geometers have written is correct. His claim that [the Pythagorean theorem] cannot be proven for a rectangle whose length is greater than its width is incorrect — it can! His words have no merit. May your peace be like a river and endure for your offspring and your name, according to your will and the will of your young student touched by your mercy. Yisrael bar Yosef.”

15. Tosafot Yom Tov (R. Yom Tov Lipmann Heller, 1578–1654), Kilayim 5:5

As I explained, with a normal vineyard planted 4 by 4, the circle 32 cubits in diameter contains exactly 45 vines. But if they are planted 5 by 5 … it contains only 37 vines. Nevertheless, since in a standard vineyard the vines are planted 4 by 4, and to the eye it’s hard to tell the difference between 4 by 4 and 5 by 5, the Sages ruled that in either case, 45 vines are ruined. For if one were to see that only 37 vines are ruined in a 5 by 5 vineyard, one might conclude that in a normal vineyard only 37 are ruined, since the difference between 4 by 4 and 5 by 5 is not clear to all. Therefore, they said that even if they’re planted 5 by 5, it ruins 45.

16. R. Abraham ibn Ezra (1089-1164), Exodus 3:15

If you make a circle whose diagonal is this number [10], and you draw a chord in a third, [the area of] the isosceles triangle will be like the circumference, and like [the area of] the big rectangle.

[Assume CE = AB/3. Can then show that area(ACD) = \(\text{area}(CEFD) = 2\sqrt{2/9} \times (AB)^2 = 0.31426968\ldots\), if AB=10, this quantity is very close to 10π.]
ישראל מסכתב להא"ן

ניבי שלום,

אני מאשר כי קיבלתי את המכתב שהועבר לי מ-irma מסכתב, ונחתם בו ימים 7.6.1940.

בשנים האחרונות התמיד נפגשתי עם מסכתב של בני משפחתו, ובראשם הנרי, שבחרوا לחיות בישראל. הנרי פרנס בא לטבריה ואחר כך בלוד שם התגורר הועבר אל ביתו של יצחק בסכר. בניהו הוא היה צורר בקהילה היהודית והיה פעיל במשטרת ישראל.intosh כי הנרי נפל בשבי הלגיון הגרמני ונהרג במצחו במהלך המערכה על בריצ'ל.

אני מצטער על מותו של הנרי ויתרון מקום מסוים במרחבי החיה היהודית. הוא היה עותק ל עולה לישראל ותרם רבות לᨔים היהודיים. אני מקווה שה䂵ת הוא יוחנן, מנהל בנק, ימשיך בProtokoller花纹 של הנרי, ויביא יבアクセק לישראל.

אני מצטער כי איני יכול לשדר את התוכן המדויק של המכתב但他 מ-irma מסכתב, אך הוא מתייחס לכניסה של בני משפחתו לישראל, ובראשם הנרי, שהורה להם להיות בארץ. אני מקווה שה requestOptions יתאימו למסכתב ול.copyOfו המדויק.

בברכה,

[חתימה]

[תאריך]

[اعتقال]