Jewish Geometry
Adam Simon Levine
NHC Summer Institute 2011

Session 1: The Diameter and Circumference of a Circle

1. I Kings 7:23–26

23. Then he made the molten sea, ten cubits from brim to brim, completely round. Its height was 5 cubits, and a line of 30 cubits went around it. 24. There were gourds below the brim, completely encircling it—ten to the cubit, encircling the sea; the gourds were in two rows, cast in one piece with it. 25. It stood upon twelve oxen: three facing north, three facing west, three facing south, and three facing east, with the tank resting upon them; their haunches were all turned inward. 26. Its thickness was a handbreadth, and its brim was made like the brim of the cup of a lily; it contained 2000 bat.

2. Ralbag to I Kings 7:23 (R. Levi b. Gershon, Spain/France, 1288–1344)

This is an approximation, for the circumference of a circle is greater than three times the circumference by approximately a seventh of the diameter. And if we say that the measurement of the circumference was taken on the inside of the basin, this is closer to the truth, but still just a close approximation. The thickness is a handbreadth, so the inner diameter is ten cubits less a third, and the inner circumference is closer to 31 1/3 cubits.
The cross-beam of which they [the rabbis] spoke must be wide enough to hold an ariah, which is half of a brick of three handbreadths. It is sufficient for a beam to be one handbreadth wide in order to hold the width of an ariah. [It must be] wide enough to hold an ariah and strong enough to support such an ariah. R. Judah ruled: [It is valid if it is] wide but not strong. If it was made of straw or reeds, we view it as though it had been made of metal; [if it was] curved it is looked upon as though it were straight; [if it was] round we view it as though it were square. Whatever has a circumference of three handbreaths has a diameter of one handbreadth. (Alternate version: If it has a circumference of three handbreaths...)

“If it is round we view it as if it were square.” Why do I need this? The next part is necessary for it: “Whatever has a circumference of three handbreaths is one handbreadth in diameter.” Whence these words? R. Johanan replied: Scripture stated: “And he made the molten sea, ten cubits from brim to brim, completely round. Its height was five cubits, and a line of thirty cubits went around it.” (II Kings 7:23) But what about its brim? R. Papa replied: Its brim was [as thin] as the flower of a lily, as it is written: “It was a handbreadth thick, and its brim thereof was wrought like the brim of a cup, like the flower of a lily; it held two thousand bat.” (7:26) But isn’t there a little? — When it was computed, it was that of the inner circumference.
5. Tosafot haRosh 2:19 (R. Asher ben Yechiel, Germany/Spain, 1250–1327)

“Whatever has a circumference of three has a diameter of one handbreadth. Whence these words?” I am baffled why it is necessary to ask this question regarding something that is clear to the eyes. A person can get an object that is one handbreadth wide and measure its circumference! It seems to me that it is meant to explain that the measurement is not exact, since the circumference is greater than three handbreadths. So the Talmud asks “Whence these things?” to permit the Sages to give a bound even though it’s not exact, but it’s close to being exact. They bring a proof from Scripture that the Solomon’s sea was the same: it was ten cubits in diameter, yet Scripture says that a line of thirty cubits encircled it, even though it was actually longer. And the Talmud objects, “What about its width?” For when Scripture says that a line of thirty cubits encircled it, it includes the brim, and therefore the line was longer than thirty cubits. And how can the verse say something that’s so contestable? And it further asks “But surely there was something?” Even though it was like a lily, they still object that it was greater than thirty cubits.


What is the flat one [i.e., the circle]? That is one that is put down [flatly] on the ground like a circular field or a circular figure. If one wants to measure [the area], let him
multiply the thread [diameter] into itself and throw away from it the one seventh and the half of a seventh; the rest is the area, its roof. And if you want to compute the area, take half of the circumference, which is 11, and multiply it into half the thread which is 3 and a half, and it amounts to 38 and a half. It is the same [result] according to the first [method] and the last one.

Now it is written: “And he made the molten sea of ten cubits from brim to brim, round in compass,” and [nonetheless] its [circumference] is thirty cubits, for it is written: “And a line of thirty cubits compassed it round about.” What is the meaning of the verse “and a line of thirty cubits...”? Since the people of the world say that the circumference of a circle contains three times and a seventh of the tread, take off from that one seventh for the thickness of the sea on the two brims, then there remain “thirty cubits compass it round about.” The seas, reservoirs, and cistern are equally [computed] according to this measure in length, breadth, and depth. Thus you have learned the measure of the circular figure.

R. Johanan said: If a sukkah was [round] like a furnace, if twenty-four men can sit around its circumference it is valid, and otherwise it is invalid. According to whom is this view? Obviously according to Rabbi, who says that a sukkah which is not four cubits square is invalid.

But consider: A man occupies the space of a cubit, and whatever has a circumference of three handbreadths has a diameter of one handbreadth. Twelve should be enough! — That applies in the case of a circle, but in the case of a square, a greater perimeter is required.

But consider: By how much is a square greater than a circle? By a quarter. Sixteen should be enough! — That applies in the case of a circle inscribed within a square, but a square inscribed within a circle a greater circumference requires more on account of the projection of the corners.

7. Babylonian Talmud, Sukkah 7b–8b

R. Johanan said: If a sukkah was [round] like a furnace, if twenty-four men can sit around its circumference it is valid, and otherwise it is invalid. According to whom is this view? Obviously according to Rabbi, who says that a sukkah which is not four cubits square is invalid.

But consider: A man occupies the space of a cubit, and whatever has a circumference of three handbreadths has a diameter of one handbreadth. Twelve should be enough! — That applies in the case of a circle, but in the case of a square, a greater perimeter is required.

But consider: By how much is a square greater than a circle? By a quarter. Sixteen should be enough! — That applies in the case of a circle inscribed within a square, but a square inscribed within a circle a greater circumference requires more on account of the projection of the corners.
But consider: If the side of a square is a cubit, its diagonal is approximately one and two fifths cubits. Sixteen and four fifths [cubits] should be enough! — [R. Johanan] gave only an approximate figure.

We could say that [R. Johanan] gave an approximate figure when the discrepancy is small, but could we make such an assumption when the discrepancy is big?! — Mar Kashisha the son of R. Hisda said to R. Ashi: Do you think that a man occupies one cubit? [The fact is that] three men occupy two cubits. How much is it [the circumference of the circle]? Sixteen cubits; and we demand here sixteen and four fifths, but [R. Johanan] gave only an approximate figure.

We could say that [R. Johanan] gave an approximate figure only for a stringency, but could we make such an assumption for a leniency?! — R. Assi answered R. Ashi: In truth, a man occupies a cubit-space, but R. Johanan does not include the space occupied by the men. How much is this? Eighteen; and sixteen and four-fifths suffice. Thus he only gave an approximate figure, in the direction of stringency.

The rabbis of Caesarea (some say: the judges of Caesarea) say: A circle inside a square is [less by] quarter, and a square inside [that] circle is [less by] half. But that’s not relevant, since it can’t be that much.

8. Tosafot, Sukkah 8a

“By how much is [the area of] a square bigger than [the area of] a circle? By a quarter.” We cannot show this from the fact that a square of 3 by 3 has a circumference of 12, and a circle of 3 has a circumference of 9, for whatever has a circumference of three has a diameter of one, as it says in our teaching. We don’t bring a proof from the fact that the circumference that is bigger by a quarter. Because of the width of a disk of 4 by 4 cubits, you might think that it only holds as much as a 3 by 3 square, since the circumference is the same. But if you divide a 3 by 3 square into 3 strips lengthwise, and 3 widthwise, you’ll find that it’s only 9 square cubits. And a round disk of 4 by 4 has an area of 12. For if a square of 4 by 4 is divided into 4 pieces lengthwise and widthwise, it will have 16 square cubits. And a square is bigger than a circle by a quarter, so you see that the circle has an area of 12. There is really no proof from the circumference at all. And also know that a strip that is 5 by 1 has a perimeter of 12, and when you divide it into 1 by 1 pieces, it has only five. And this is the reason: when you put a thread on a square, it makes corners, and when you put it on a circle, it spreads out.

And if we come to make precise the calculation of a square bigger than a circle, we will be able to prove it as follows. Make a division [of the radius] of very small distances, and trace it around with many threads, one around another until you fill up the circle. And the width grows handbreadth by handbreadth. And then you cut the threads off of the division. And below, there’s half the diameter of the circle, and after you cut, you lay out all the threads from right to left. And you find that every thread goes and extends from its neighbor, some from here and some from here, until you arrive at the top thread, whose length is 3 handbreadths, since it’s the outside of a circle of one handbreadth by one handbreadth, and anything that has a width of one has a circumference of three. These threads are found arranged in this manner, like a kind of
strip, its width in the middle is half a handbreadth, and at each point, from here and from here it narrows down to nothing. And if you cut it in the middle, you find two pieces, each with a length of one and a half handbreadths and a width of half a handbreadth, and one side it narrows down to nothing. And now put together these two pieces, and put the long one opposite the short one. You will find a strip that is one and a half handbreadths long and half a handbreadth wide. Divide this into three pieces. You will find three pieces of half a handbreadth by half a handbreadth. And if you have a square of one handbreadth, when you divide it you'll find four pieces of half a handbreadth by half a handbreadth. Behold, a square is bigger than a circle by a quarter!
You brought a proof from the language of the Talmud, which says: “He was imprecise for a stringency.” They didn’t say this because of this geometric precision, but rather even in their essence they were stringent that it wasn’t precise. As it says in the first chapter of Sukkah (7b) about a sukkah in the shape of a furnace. That discussion depends on two principles: Whatever has a circumference of three handbreadths has a diameter of one handbreadth, and a square of one cubit has a diagonal of a cubit and two fifths, which requires R. Johanan to say that the circumference of the sukkah must seat 24 people in accordance with his assumption that any sukkah that isn’t four cubits by four cubits is impermissible. If the circumference seats 24 people, and a person sits in a cubit, and the people are outside, then the diameter is eight cubits since its circumference is 24. And when you take away two cubits for the person on each side, six cubits are left, and we only need five and three fifths, since a square of one cubit has a diagonal of a cubit and three fifths. And R. Johanan was strict in requiring six cubits. That’s the summary of their words, even though they didn’t say it in that language.

And if the sages were worried about these geometric precisions, as you say, and they were trying to correct the statement of R. Johanan to make it close to the truth, they should have mentioned this here and said that even though, according to the principles that we laid out, there are in R. Johanan’s words an extra two fifths, yet in truth there is no noticeable excess. For in a square of one cubit, the diagonal is more than a cubit and two fifths, and a circle one handbreadth in diameter has a circumference of more than three handbreadths. And in this, the words of R. Johanan are amazingly close to the truth. For a sukkah with a circumference of 24 cubits has a diameter of only 7 2/3 cubits, roughly, and when you take away from this two cubits because of the people, we’re left with 5 2/3 cubits, and the diagonal of a four-cubit square is 5 3/5 cubits, roughly. And the words of R. Johanan come to be very close to the knowledge of the geometers, for 3/5 is close to the increase. It’s bigger than nine and two thirds, roughly, in the plane. It’s smaller than 15. And the two figures are equal, so they didn’t mention in the Gemara that they were trying to fix his words to be close to the truth. And it seems that they weren’t at all worried about this geometric precision…

We could say one of two things: One, this could just be the Sages’ received tradition, to work in this manner. And even though it’s an approximation, these measures are halacha given to Moses at Sinai. And we could say that this is how halacha given to Moses at Sinai is stated. The Torah was not given to the ministering angels! And perhaps the halacha transmitted to them was that they should work with these principles, even though they’re approximate, as if they were precise. And there is support from this from the sea of Solomon, which Scripture describes with an approximation, as I’ve explained. This is one explanation. Or one could say that they used these values to instruct students, as it says: “One should always teach one’s student in the easiest way” (Pesachim 3b, 63b). But for actual use, we should use the precise, true figure, which was passed to the Sages by those who knew the measures. It seems the halacha is taught to beginning students and the actual use is taught to wise ones in order to be precise about the true value. And I think this is the correct way to interpret the Sages’ words.
You need to know that the ratio of the circle’s diameter to its circumference is not known, and it is never possible to express it precisely. This is not due to a lack in our knowledge, as the fools think, but it is in its nature that it is unknown, and there is no way to know it. But it can be measured approximately, and the geometers have already written essays about this, that is, to know the ratio of the diameter to the circumference approximately, and the proofs for this. The approximation which is accepted by the educated people is the ratio of one to three and one seventh. Every circle whose diameter is one handbreadth, has in its circumference three and one seventh handbreadths approximately. As it will never be perceived but approximately, they took the nearest integer and said that every circle whose circumference is three fists is one fist wide, and they contented themselves with this for their needs in the religious law.


We must now ask: Why is the sea described in Kings in such a way that seems to the reader to be so imprecise? Must one who reads the verse there think that the circumference of the sea was only thirty cubits and not realize that the prophet knew that its circumference was more than 31 cubits in reality, and that the figure thirty cubits was given only for halacha? Is there any sign in the verse of our precision? Let us look carefully at the above-mentioned verse, on which the figure of 3:1 is based. After it says “perfectly round,” it mentions the height of the sea, and then adds: “And a line of thirty cubits encircled it all around.” One may ask: Why doesn’t it say, more briefly, “It was round, thirty cubits around” instead of the long version: “And a line of thirty cubits encircled it all around?” It seems to me that the author chose this long phrase because he wanted to add the word וְ, which has both a pronounced (k’ri) and a written (k’tiv) form. The written form is והוּה, and the pronounced form is והו. What can we learn from the pronounced and written forms? When we are speaking about numbers, we explain that the difference between the versions can be found in a numerical manner, in gematria. The \textit{gematria} of והו (the conjunctive \textit{vav} isn’t part of the word) is 111. The \textit{gematria} of והו is 106. A mathematical calculation of the circumference of the sea, based on a diameter of 10 cubits, equals $10 \times \pi = 31.4159265\ldots$. If we use the estimate $3330/106$, we obtain a
number that is closer to the correct number than the number obtained using 220/7 as usual. The ratio 330/106 : 30 is equal to the ratio 111 : 106, the *gematria* values as above of קוה and קו. The closeness is so surprising as to amaze.

If we compare 31.4159265 : 30 = 1.0471975 to 111:106 = 1.0471698, we see that the difference is smaller than three parts in 100,000, an extremely good estimate.

What did the prophet want to teach us with these written and pronounced forms? The written form serves as a sign of the exact value of the circumference, and the pronounced form teaches us what we need to know for *halacha*. The written form is what we see with our physical eyes, and therefore it is connected to the number that we find if we measure the circumference. The pronounced value is what gives us knowledge for matters of *halacha*. If we explain the written form in this manner, we cannot say that the prophet did not give us the exact value of [the circumference of] the sea properly in his statement that “a line of thirty cubits encircled it all around.” Really, he is telling us the circumference in accordance with our reality, albeit only with a hint; his main intention was to teach us *halacha*.

One may ask: Why isn’t the word is קוה written with the same written and pronounced forms in the other verse as well (II Chronicles 4:2)? We would answer, that there is no need to repeat the sign of what is in accordance with our reality. And one may ask: Don’t we also find written and pronounced forms of קוה in two other places in the Bible: “And the line will go straight out” (Jeremiah 31:38), and “The line is being applied to Jerusalem” (Zechariah 1:16)? We answer that we do not have to explain all written and pronounced forms in the same way.


That circle—which, begotten so, appeared in You as light reflected—when my eyes had watched it with attention for some time, within itself and colored like itself to me seemed painted with our effigy, so that my sight was set on it completely. As the geometer intently seeks to square the circle, but he cannot reach, through thought on thought, the principle he needs, so I searched that strange sight: I wished to see the way in which our human effigy suited the circle and found place in it—and my own wings were far to weak for that. But then my mind was struck by light that flashed And, with this light, received what it had asked. Here force failed my high fantasy; but my Desire and will were moved already—like a wheel revolving uniformly—by the Love that moves the sun and the other stars.