1. Introduction

My research focuses on invariants of knots, links, and graphs in 3-manifolds. Knots and links play a key role in the study of 3-manifolds. Indeed, one can show that any 3-manifold can be constructed by performing a simple operation known as Dehn surgery on a suitable link in $S^3$. One can consider knots in any 3-manifold, but so far most work on knot invariants has concentrated on knots in $S^3$. For example, two important knot invariants are Khovanov homology [18] and singular instanton homology [21, 22]. Khovanov homology is easy to calculate (it is the homology of a chain complex constructed from a generic planar projection of a knot), but is defined only for knots in $S^3$. Singular instanton homology is difficult to calculate (it is defined in terms of gauge-theoretic nonlinear PDE’s) and, though it is defined for knots in arbitrary 3-manifolds, it has been calculated only for some knots in $S^3$. Little is known about invariants of knots in arbitrary 3-manifolds, but, partly because of the close connection between knots and 3-manifold topology, such results would be of great interest.

One focus of my research is on generalizing Khovanov homology to knots in lens spaces. I have considered two different strategies for achieving this goal. In [8], I described a strategy based on generalizing Seidel and Smith’s interpretation of Khovanov homology as Lagrangian Floer homology in symplectic manifolds known as Seidel-Smith spaces [28]. As a first step, I proposed candidate manifolds that generalize the Seidel-Smith spaces to the case of knots in lens spaces. These candidate manifolds are related to moduli spaces of Hecke modifications of holomorphic vector bundles over an elliptic curve, and the project involved a detailed study of these moduli spaces. In [10], I used the Hecke-modification techniques I developed in [8] to explicitly describe a moduli space of stable parabolic bundles over an elliptic curve. Some of my work on Hecke modifications has also been applied to the geometric Langlands program [11].

In [9], I described a second strategy for constructing Khovanov homology for knots in lens spaces based on generalizing a symplectic interpretation of Khovanov homology for knots in $S^3$ due to Hedden, Herald, Horgancamp, and Kirk [16]. The strategy relies on a partly conjectural description of the Fukaya category of the traceless $SU(2)$ character variety $R^+(T^2, 2)$ of the 2-torus with two punctures. From a diagram of a 1-tangle in a solid torus, I constructed a corresponding object $(X, \delta)$ in the $A_{\infty}$ category of twisted complexes over this Fukaya category. The homotopy type of $(X, \delta)$ is an isotopy invariant of the tangle diagram. I used $(X, \delta)$ to explicitly construct chain complexes for knots in $S^3$ and some knots in $S^2 \times S^1$. For knots in $S^3$, the cohomology of the chain complex reproduces Khovanov homology, as I showed in [7], though the chain complex itself is not the usual one. For knots in $S^2 \times S^1$, I proved results that suggest the cohomology of the chain complex may be a knot invariant.

In related work, in [6] I generalized ideas of Hedden, Herald, and Kirk [14] to compute generating sets for the singular instanton homology of knots in lens spaces. The generating sets are given by the intersection points of two Lagrangians in $R^+(T^2, 2)$ (conjecturally, the Lagrangian Floer homology of the Lagrangians gives the singular instanton homology itself). I computed generating sets for several example knots, some of which produce known results for knots in $S^3$ and some of which provide original results for knots in lens spaces.
To define invariants of knots, it is often useful to generalize from knots to graphs, also called webs, which can be thought of as knots with singular behavior allowed at the vertices. For example, webs arise naturally in knot invariants defined via representation theory, since they can be viewed as pictorial descriptions of rules for combining representations. To categorify such invariants, it is useful to define a category in which the objects are webs and the morphisms are web cobordisms called foams. In fact, Khovanov homology can be viewed as just such a categorification [3]. Webs also play an important role in a new strategy for proving the four-color theorem due to Kronheimer and Mrowka [26]. Kronheimer and Mrowka use a version of singular instanton homology to define a functor \( J^2 \) from a category of foams to the category of vector spaces over \( \mathbb{F} \), the field of two elements. They show that if the dimension of the vector space \( J^2(K) \) associated to a web \( K \) is equal to the number of 3-colorings \( \text{Tait}(K) \) of \( K \) for all webs \( K \), this would imply the four-color theorem. They also show that \( \dim J^2(K) \) and \( \text{Tait}(K) \) are in fact equal for a special class of “reducible” webs.

In [5], I investigated a possible combinatorial replacement \( J^p \) for \( J^2 \) that was defined by Kronheimer and Mrowka. I wrote a computer program to calculate lower bounds on \( \dim J^p(K) \) and calculated such bounds for a number of example nonreducible webs. In some cases the bounds are sufficiently strong to uniquely determine \( \dim J^p(K) \), and these results constitute the first exact calculations of \( \dim J^p(K) \) for nonreducible webs.

2. Khovanov homology for knots in lens spaces, strategy I

In [8] I considered the problem of generalizing Khovanov homology to knots in lens spaces. Since Khovanov’s original combinatorial definition of Khovanov homology [18] does not suggest an obvious generalization, my strategy was to first reinterpret Khovanov homology in geometric terms using ideas due to Seidel and Smith. Seidel and Smith describe a knot in \( S^3 \) as the closure of a braid with \( 2m \) strands. Such a braid can be obtained by gluing together two solid balls, each containing \( m \) unknotted arcs, along their common \( S^2 \) boundary. To the 2-sphere they associate a symplectic manifold \( \mathcal{Y}(S^2, 2m) \) known as the Seidel-Smith space, and to the pair of solid balls they associate a pair of Lagrangians in \( \mathcal{Y}(S^2, 2m) \). Seidel and Smith prove that the Lagrangian Floer homology of the pair of Lagrangians is a knot invariant that they conjectured would coincide with the Khovanov homology of the knot [28]; this was later proved by Abouzaid and Smith [1].

One might hope that a similar picture holds for knots in lens spaces. Given a suitable Heegaard splitting of the lens space into two solid tori, perhaps one can associate to their common \( T^2 \) boundary a symplectic manifold \( \mathcal{Y}(T^2, 2m) \) that generalizes the Seidel-Smith space \( \mathcal{Y}(S^2, 2m) \), and to the pair of solid tori a pair of Lagrangians in \( \mathcal{Y}(T^2, 2m) \). If the Lagrangian Floer homology of the pair of Lagrangians could be shown to be independent of the Heegaard splitting, it would serve as a natural generalization of Khovanov homology.

The first step in implementing this strategy is to find a suitable symplectic manifold \( \mathcal{Y}(T^2, 2m) \), and in [8] I proposed a natural candidate for this space. The space \( \mathcal{Y}(S^2, 2m) \) was originally defined in terms of a nilpotent slice in the Lie algebra \( \mathfrak{sl}_{2m} \). This description of \( \mathcal{Y}(S^2, 2m) \) does not seem to have a natural generalization, so I first applied a result of Kamnitzer [17] to reinterpret \( \mathcal{Y}(S^2, 2m) \) as a moduli space \( \mathcal{H}(\mathbb{CP}^1, 2m) \) of Hecke modifications of rank 2 holomorphic vector bundles over a rational curve; roughly, this space parameterizes different ways of locally modifying a given vector bundle so as to obtain a new vector bundle. Next, I proposed a notion of a Hecke modification of a parabolic bundle and used this notion to reinterpret the moduli space \( \mathcal{H}(\mathbb{CP}^1, n) \) of Hecke modifications of vector bundles as a moduli space \( \mathcal{P}(\mathbb{CP}^1, n) \) of Hecke modifications of parabolic bundles. I showed that \( \mathcal{P}(\mathbb{CP}^1, n) \) has a natural generalization \( \mathcal{P}(X, n) \) to the case of an elliptic curve \( X \), and I proposed the space \( \mathcal{P}(X, 2m) \) as a candidate for \( \mathcal{Y}(T^2, 2m) \). In summary, the sequence of reinterpretation and generalization is

\[
\mathcal{Y}(S^2, 2m) \cong \mathcal{H}(\mathbb{CP}^1, 2m) \cong \mathcal{P}(\mathbb{CP}^1, 2m) \quad \rightsquigarrow \quad \mathcal{P}(X, 2m) \rightleftharpoons \mathcal{Y}(T^2, 2m).
\]
In fact, I can define the moduli space $\mathcal{P}(C,n)$ for an arbitrary algebraic curve $C$. I conjecture that $\mathcal{P}(C,n)$ canonically embeds into the moduli space $M^s(C,n+3)$ of stable rank 2 parabolic bundles over $C$ with $n+3$ marked points, and I proved this for $\mathcal{P}(\mathbb{C}P^1,n)$ and $\mathcal{P}(X,n)$:

**Theorem 2.1.** The space $\mathcal{P}(\mathbb{C}P^1,n)$ canonically embeds into the moduli space $M^s(\mathbb{C}P^1,n+3)$.

**Theorem 2.2.** The space $\mathcal{P}(X,n)$ canonically embeds into the moduli space $M^s(X,n+3)$.

Theorem 2.1 appears to be closely related to a theorem due to Woodward [29] that was proven using the nilpotent slice interpretation of $\mathcal{Y}(S^2,2m)$:

**Theorem 2.3** (Woodward). The space $\mathcal{Y}(S^2,2m)$ embeds into the moduli space $M^s(\mathbb{C}P^1,2m+3)$.

These embedding results suggest that there may be a spectral sequence from symplectic Khovanov homology to symplectic singular instanton homology, which is defined in terms of $M^s(C,2m+3)$, that generalizes a spectral sequence due to Kronheimer and Mroka from Khovonov homology to singular instanton homology [23]. The embedding results thus provide some additional evidence that $\mathcal{P}(X,2m)$ is the correct generalization of the Seidel-Smith space $\mathcal{Y}(S^2,2m)$.

In order to define and characterize the moduli space $\mathcal{P}(X,n)$, I described all possible Hecke modifications of all possible rank 2 vector bundles over an elliptic curve $X$, using a classification of holomorphic vector bundles on elliptic curves due to Atiyah [2]. As an additional application of these results, in [10] I used Hecke-modification methods to describe the moduli space $M^s(X,3)$:

**Theorem 2.4.** The moduli space $M^s(X,3)$ is a blow-up of an embedded elliptic curve in $(\mathbb{C}P^1)^3$.

In future work, I would like to continue to pursue this strategy of generalizing Khovanov homology to lens spaces. It would also be of interest to use the embedding results to attempt to construct a spectral sequence to symplectic singular instanton homology.

3. Khovanov homology for knots in lens spaces, strategy II

In [9] I consider a second strategy for generalizing Khovanov homology to knots in lens spaces inspired by a symplectic interpretation of Khovanov homology for knots in $S^3$ due to Hedden, Herald, Hogancamp, and Kirk [16]. Given a link $L$ in $S^3$, they consider a Heegaard splitting

$$(S^3,L) = (B^3,T_1) \cup_{(S^2,4)} (B^3,T_2)$$

such that the Heegaard surface $(S^2,4)$ is a 2-sphere that transversely intersects $L$ in four points, the handlebodies $(B^3,T_1)$ and $(B^3,T_2)$ are closed 3-balls containing 2-tangles $T_1$ and $T_2$, and $T_1$ is trivial. To the Heegaard surface $(S^2,4)$ they associate the traceless $SU(2)$ character variety $R^*(S^2,4)$ for the 2-sphere with four punctures, a symplectic manifold known as the pillowcase. They project the tangle $T_2$ onto the plane to obtain a 2-tangle diagram $T$ in the disk. From a cube of resolutions of $T$, they construct an object $(X,\delta)$ in the $A_\infty$ category of twisted complexes over the Fukaya category of $R^*(S^2,4)$, where $X$ consists of shifted copies of Lagrangians corresponding to planar tangles at the vertices of the cube and $\delta : X \to X$ consists of maps of Lagrangians corresponding to saddles at the edges of the cube. They prove:

**Theorem 3.1.** (Hedden–Herald–Hogancamp–Kirk) Given a planar projection of $(B^3,T_2)$, there is a corresponding object $(X,\delta)$ of the twisted Fukaya category of $R^*(S^2,4)$. The homotopy type of $(X,\delta)$ is an isotopy invariant of $T_2$ rel boundary.

To the trivial tangle $T_1$ they assign an object $(W_1,0)$ of the twisted Fukaya category. The morphism spaces of the twisted Fukaya category have the structure of cochain complexes, so in particular the space of morphisms from $(W_1,0)$ to $(X,\delta)$ is a cochain complex. They show this cochain complex is the usual cochain complex for the reduced Khovanov homology $Khr(L)$ of $L$, thus proving:
Theorem 3.2. (Hedden–Herald–Hogancamp–Kirk) We have an isomorphism of bigraded vector spaces $Khr(L) \to H^*(\text{hom}((W_1,0),(X,\delta)))$.

Theorem 3.2 shows that the Fukaya category of $R^*(S^2,4)$ knows about Khovanov homology. My strategy for generalizing Khovanov homology to links in lens spaces is based on generalizing this observation. Theorem 3.2 is formulated in terms of $R^*(S^2,4)$ because this is the character variety corresponding to the Heegaard surface $(S^2,4)$ for the chosen Heegaard splitting of $(S^3,L)$. More generally, given a link $L$ in a lens space $Y$, one can consider a Heegaard splitting $(Y,L) = (U_1,T_1) \cup (U_2,T_2)$ such that the Heegaard surface $(T^2,2)$ is a 2-torus that transversely intersects $L$ in two points, the handlebodies $(U_1,T_1)$ and $(U_2,T_2)$ are solid tori containing tangles $T_1$ and $T_2$, and $T_1$ is trivial. To the Heegaard surface $(T^2,2)$ I associate the traceless $SU(2)$ character variety $R^*(T^2,2)$ for the 2-torus with two punctures. I project the tangle $T_2$ onto the plane to obtain a 1-tangle diagram $T$ in the annulus. Based on a detailed study of the Fukaya category of $R^*(T^2,2)$, supplemented with some conjectures regarding certain $A_\infty$ operations of this category that I have not calculated explicitly, I generalize Theorem 3.2 for $R^*(S^2,4)$:

Theorem 3.3. If the $A_\infty$ operations of $R^*(T^2,2)$ are as conjectured, then given a planar projection $T$ of $(U_2,T_2)$ there are two corresponding objects $(X,\delta_+)$ and $(X,\delta_-)$ of the twisted Fukaya category of $R^*(T^2,2)$. The homotopy type of $(X,\delta_\pm)$ is an invariant of $T_2$ rel boundary.

Given a 1-tangle diagram $T$ in the annulus, one can close $T$ with an overpass arc $A_+$ or underpass arc $A_-$ to obtain a diagram of a link $L_{T_+}$ or $L_{T_-}$ in $S^3$. I use the twisted complex $(X,\delta_\pm)$ to explicitly construct a cochain complex $(C_\pm,\partial_\pm)$ for the link $L_{L_{T\pm}}$ from a cube of resolutions of $T$ and prove the following generalization of Theorem 3.2:

Theorem 3.4. There is an isomorphism of bigraded vector spaces $Khr(L_{T\pm}) \to H^*((C_\pm,\partial_\pm))$.

One can also close $T$ to obtain a diagram of a link $L_{T^0}$ in $S^2 \times S^1$. For certain tangle diagrams, I use the twisted complex $(X,\delta_\pm)$ to explicitly construct a cochain complex $(C_0,\partial_0)$ for the link $L_{T^0}$ from a cube of resolutions of $T$. One might hope that the cohomology of $(C_0,\partial_0)$ depends only on the isotopy class of the link $L_{T^0}$ and not on its description as the closure of the particular tangle diagram $T$. From Theorem 3.3 it follows that the cohomology is an isotopy invariant of the tangle diagram $T$. But it is possible for nonisotopic tangle diagrams $T_1$ and $T_2$ to yield isotopic links $L_{T^0}$ and $L_{T^2}$ in $S^2 \times S^1$, and example calculations show that in this situation the cohomology for $T_1$ and $T_2$ need not be the same. In all the examples I have checked, however, the dependence on the tangle diagram is reflected only in the bigradings of generators, and the cohomology does agree if the bigradings are collapsed from $\mathbb{Z}$ to $\mathbb{Z}_2$. Such examples motivate:

Conjecture 3.1. The cohomology of the cochain complex $(C_0,\partial_0)$ with bigradings collapsed from $\mathbb{Z}$ to $\mathbb{Z}_2$ is an isotopy invariant of the link $L_{T^0}$.

I have proven a partial invariance result in support of Conjecture 3.1 as described in [9]. In future work, it would be interesting to use these methods to construct and investigate cochain complexes for links in other lens spaces.

4. Singular instanton homology for knots in lens spaces

In related work, in [6] I used the character variety $R^*(T^2,2)$ to construct generating sets for the singular instanton homology of knots in lens spaces. This work generalizes a scheme due to Hedden, Herald, and Kirk for constructing generating sets for the singular instanton homology of knots in $S^3$ [14] [15]. Roughly speaking, the singular instanton homology of a knot $K$ in a 3-manifold $Y$ is a kind of Morse homology in which the Chern-Simons functional serves as a Morse function on the
space of connections on a trivial $SU(2)$ bundle. The generators of the homology chain complex correspond to gauge orbits of flat connections with prescribed singularities along $K$, which in turn correspond to conjugacy classes of homomorphisms $\pi_1(Y - K) \to SU(2)$ that take loops around $K$ to traceless matrices. The set of such conjugacy classes defines a character variety $R(Y, K)$.

The character variety $R(Y, K)$ is not quite the generating set that we seek, since there are two technical complications that must be addressed. First, in order to get a chain complex, with $\partial^2 = 0$, we need to ensure there are no “reducible” connections, which correspond to abelian $SU(2)$ representations. Second, the Chern-Simons functional is generally not Morse, and must be perturbed so as to render it Morse. The modifications needed to address these issues amount to considering homomorphisms $\pi_1(Y - K') \to SU(2)$ that satisfy certain conditions, where $K'$ is a graph containing $K$ as a subset whose precise form I describe. These homomorphisms define a modified character variety $R^2(Y, K)$ that constitutes the actual generating set for singular instanton homology.

To compute $R^2(Y, K)$, I Heegaard-split $(Y, L)$ as described in equation (1). The solid torus $U_1$ contains a portion of the knot consisting of a trivial 1-tangle $T_1$, together with all of the modifications necessary to eliminate reducible connections and to render the Chern-Simons functional Morse. The solid torus $U_2$ contains a 1-tangle $T_2$ that describes the remainder of the knot. In analogy with $R(Y, K)$ and $R^2(Y, K)$, I define character varieties $R^2(U_2, T_2)$ and $R^2(Y, L)$. Pulling back homomorphisms along inclusion maps, we obtain the following commutative diagram:

$$
\begin{array}{ccc}
R^2(U_1, T_1) & \to & R^2(Y, L) \\
\downarrow & & \downarrow \\
R^*(U_2, T_2) & \to & R^*(T^2, 2)
\end{array}
$$

The character variety $R^*(T^2, 2)$ is a symplectic manifold, and the images $L_1$ and $L_2$ of $R^2(U_1, A_1)$ and $R^2(U_2, A_2)$ in $R^*(T^2, 2)$ are Lagrangians. I prove theorems that explicitly describe the character variety $R^2(T^2, 2)$ and the Lagrangians $L_1$ and $L_2$. In particular, the Lagrangian $L_1$ is an immersed 2-sphere with a unique double point. I prove:

**Theorem 4.1.** A point $[\rho] \in L_1 \cap L_2 \subset R^*(T^2, 2)$ that is not the double point of $L_1$ is the image of a unique point $[\rho'] \in R^2(Y, K)$ under the pullback map $R^2(Y, K) \to R^*(T^2, 2)$. The point $[\rho']$ is nondegenerate if and only if the intersection of $L_1$ with $L_2$ at $[\rho]$ is transverse.

For simplicity I have stated Theorem 4.1 only for the important special case that $T_2$ is an unknotted arc, so $K$ is a $(1,1)$-knot, but I also proved a version of Theorem 4.1 that does not require this assumption. From Theorem 4.1 it follows that if $L_2$ intersects $L_1$ transversely and does not contain the double-point of $L_1$, then every point in $R^2(Y, K)$ is nondegenerate and the pullback map $R^2(Y, K) \to R^*(T^2, 2)$ is injective with image $L_1 \cap L_2$. Thus $R^2(Y, K)$ is a generating set for singular instanton homology consisting of $|L_1 \cap L_2|$ generators.

I explicitly calculate generating sets for the singular instanton homology of several example $(1,1)$-knots. In particular, one can define the notion of a “simple” knot in a lens space [13], and I prove the following result:

**Theorem 4.2.** If $K$ is the unique simple knot in the lens space $L(p, 1)$ representing the homology class $1 \in \mathbb{Z}_p = H_1(L(p, 1); \mathbb{Z})$, then the rank of the singular instanton homology of $K$ is at most $p$. 
For a simple knot $K$ in the lens space $Y = L(p, q)$, the knot Floer homology $\widehat{HFK}(Y, K)$ has rank $p$ [13]. Thus, Theorem 4.2 is consistent with Kronheimer and Mrowka’s conjecture that for a knot $K$ in a 3-manifold $Y$, the ranks of singular instanton homology and knot Floer homology $\widehat{HFK}(Y, K)$ are the same [20]. In future work, it would be interesting to test Kronheimer and Mrowka’s conjecture for additional (1,1)-knots in $S^3$. There is a combinatorial method for computing the knot Floer homology for such knots [12].

5. Computer bounds for Kronheimer-Mrowka foam evaluation

Kronheimer and Mrowka have recently described a new strategy for proving the four-color theorem without the aid of computers [26]. Their strategy involves a category Foams whose objects are webs, which are unoriented planar trivalent graphs, and whose morphisms are foams, which are singular cobordisms between pairs of webs. Using a version of singular instanton homology, they define a functor $J^\sharp$ from the category Foams to the category Vect$_F$ of vector spaces over $F$, the field of two elements. They also consider a possible combinatorial replacement $J^\flat$ for $J^\sharp$. They first define $J^\flat(F) \in F$ for closed foams $F$, which are cobordisms from the empty web to itself, via a set of combinatorial rules that they conjectured were well-defined. This was later shown to be the case by Khovanov and Robert [19], who gave an explicit formula for $J^\flat(F)$. Kronheimer and Mrowka then extend $J^\flat$ to a functor by using the universal construction [4]; in particular, for a web $K$ they define the corresponding vector space $J^\flat(K)$ as follows. First, define a vector space $V(K)$ spanned by all half foams with top boundary $K$; these are cobordisms from the empty web to $K$. Define a bilinear form $(-, -) : V(K) \otimes V(K) \to F$ such that $(F_1, F_2) = J^\flat(F_1 \cup_K \overline{F}_2)$, where $F_1 \cup_K \overline{F}_2$ is the closed foam obtained by reflecting $F_2$ top-to-bottom to get $\overline{F}_2$ and then gluing it to $F_1$ along $K$. Now define $J^\flat(K)$ to be the quotient of $V(K)$ by the orthogonal complement of $V(K)$ relative to $(-, -)$.

In light of Kronheimer and Mrowka’s new strategy for proving the four-color theorem, it is important to understand the relationships between $\dim J^\flat(K)$, $\dim J^\sharp(K)$, and Tait($K$), the number of 3-colorings of the web $K$, for arbitrary webs $K$. It is known that for any web $K$ these three numbers are related by $\dim J^\flat(K) \leq \text{Tait}(K) \leq \dim J^\sharp(K)$, and for a special class of “reducible” webs $K$ these three numbers coincide ($\dim J^\flat(K) = \text{Tait}(K) = \dim J^\sharp(K)$) [5, 19, 24]. It is thus of interest to compute examples of $\dim J^\flat(K)$ and $\dim J^\sharp(K)$ for nonreducible webs $K$. The only results that have previously been obtained are for the dodecahedral web $W_1$, for which it is known that $J^\flat(W_1) \geq 58$, Tait($W_1$) = 60, and $J^\sharp(W_1) \leq 68$ [25, 26]. Since $J^\flat(K)$ is defined in terms of an infinite number of generators mod an infinite number of relations, it is not clear whether $\dim J^\flat(K)$ can be algorithmically computed. Nevertheless, it is possible to algorithmically compute lower bounds for $\dim J^\flat(K)$ by computing the rank of the restriction of the bilinear form $(-, -)$ to any finite subspace of $V(K)$.

In [5], I describe a computer program I wrote to determine such lower bounds by enumerating a large number of half-foams and computing the rank of the restriction of the bilinear form $(-, -)$ to the vector space that they span. I compute lower bounds on $\dim J^\flat(K)$ for a number of example nonreducible webs. Since $\dim J^\flat(K) \leq \text{Tait}(K)$, if the lower bound on $\dim J^\flat(K)$ is equal to $\text{Tait}(K)$ then $\dim J^\flat(K) = \text{Tait}(K)$. In this way I was able to perform the first calculations for $\dim J^\flat(K)$ for $K$ nonreducible:

**Theorem 5.1.** For the nonreducible webs $W_2$ and $W_3$ shown in Figure 2, we have that $\dim J^\flat(W_2) = \text{Tait}(W_2) = 120$ and $\dim J^\flat(W_3) = \text{Tait}(W_3) = 162$.

The vector spaces $J^\flat(K)$ carry a quantum grading, which is induced from a grading on foams, and the computer program can also find lower bounds on the quantum dimension $\text{qdim} J^\flat(K)$, which in some cases yield exact results. In particular, I calculate the quantum dimensions of the nonreducible webs $W_2$ and $W_3$. 
In future work, the computer program could be adapted to other projects involving categorification in terms of foams. For example, I could write a program to compute a knot homology theory recently proposed by Robert and Wagner [27] that categorifies the Alexander polynomial, and which they conjecture coincides with knot Floer homology. It would also be of interest to attempt to compute the dimension of $J^0(K)$ for some example nonreducible webs $K$.

References


