Titles and Abstracts:

Michael Christ: Title: Implicitly oscillatory quadrilinear integrals

Abstract: Multilinear oscillatory integrals arise in various contexts in harmonic analysis, in partial differential equations, and in ergodic theory. Inequalities for oscillatory integrals were a favorite subject of Eli Stein; they were the topic of two chapters of his 1993 book, and of multiple classic papers of Stein and collaborators. With Phong and Sturm, he did important early work on the multilinear case.

We discuss the majorization of integrals of finite products by negative order Sobolev norms of the factors, where integration is over a ball in Euclidean space and each factor is the composition of an arbitrary function with a fixed mapping to a lower-dimensional Euclidean space. The talk focuses on the case of four factors, with each codomain one-dimensional. This is related to earlier work on the trilinear case of Bourgain (1988), of Joly, Metivier, and Rauch (1995), and of the speaker (2019), as well as to more recent work of Mirek et. al.

Sublevel set inequalities, a topic in differentiable combinatorics, are a central element of the analysis. These quantify the extent to which certain systems of linear equations are unsolvable. We will state an inequality and indicate some of the main elements in its proof. The new development is the elimination of virtually all auxiliary hypotheses; the only assumptions are a natural necessary condition, and real analyticity of the mappings.

Galia Dafni: Title: Hardy spaces, BMO and some variants

Abstract: The fundamental contributions of Eli Stein to the development of the theory of real Hardy spaces and BMO need no introduction. The talk concerns some lesser-known variants of these spaces, in particular the John-Nirenberg spaces JN_p, $1 < p < \infty$, the space of functions of vanishing mean oscillation, VMO, introduced by Sarason, and the nonhomogeneous (aka “local”) Hardy spaces and bmo, introduced by Goldberg. We will describe recent work on nontriviality and duality for JN_p (joint with T. Hytönen, R. Korte and H. Yue), boundedness, continuity and dimension dependence of bounds for rearrangements on BMO and VMO (joint work with A. Burchard and R. Gibara), and extension domains for nonhomogeneous bmo and vmo (joint with A. Butaev).

Phil Gressman: Title: Analysis of smoothly-varying geometric objects

Abstract: It is a well-known and perhaps poorly-understood phenomenon that many elementary techniques and tools used in harmonic analysis (e.g., sublevel set estimates and van der Corput's lemma) work extremely well in one dimension and immediately lose much of their power and elegance in higher dimensions. Inspired by Stein's way of understanding the behavior of smooth functions and the more complicated operators they govern, we will discuss some recent results arising in the theory of L^p-improving properties of Radon-like operators as first steps toward a new quantitative understanding of smoothly-varying geometric objects like subspaces or submanifolds that respects the intrinsic affine symmetries exhibited by Radon-like operators and Fourier extension operators. The results include some unexpected connections to Brascamp-Lieb inequalities and the Phong-Stein rotational curvature condition.

Larry Guth: Title: Discussion of Montgomery's conjecture on large values of Dirichlet polynomials

Abstract: A Dirichlet polynomial is a function $D(t) = sum_{n=1}^N b_n n^{it}$. Montgomery formulated a conjecture about the L^p norms of Dirichlet polynomials. Improved estimates about this problem would imply improved estimates about the zeroes of the Riemann zeta function and the distribution of primes.

In the late 80s, Bourgain made an analogy between Montgomery's conjecture and the restriction conjecture. Notice that the Fourier transform of a Dirichlet polynomial D(t) is supported on the sequence log n, where n is an integer from 1 to N. Using this analogy, Bourgain proved that Montgomery's conjecture implies the Kakeya conjecture.

Over the last few years, I have tried to take techniques from restriction theory and apply them to the Montgomery conjecture. In particular, I tried to use techniques related to decoupling theory, where wave packets are a leading character. These techniques did not lead to any progress. I have come to realize that Montgomery's problem involves some issues that are different from the issues in the restriction problem, and that wave packet type arguments do not help. I will describe these issues. I will also discuss a possible approach to working on these issues -- although it has not yet led to any progress.

Carlos Kenig: Title: Channels of energy for wave equations, non-radiative solutions and soliton resolution

Abstract: We will discuss the role of non-radiative solutions to nonlinear wave equations, in connection with soliton resolution. Using new channels of energy estimates we characterize all radial non-radiative solutions for a general class of nonlinear wave equations. This is joint work with C.Collot, T. Duyckaerts and F. Merle.

Sergiu Klainerman: Title: Open problems on linear and nonlinear wave equations

Abstract: I plan to talk about two open problems where the type of Fourier analysis pioneered by Eli Stein is relevant. The first concerns Strichartz and bilinear estimates and their applications to the study of long time properties of quasilinear wave equations, such as those that appear in General Relativity. The second is the problem of deriving time decay estimates for solutions of linear wave equations with multiple characteristics typical to crystal optics and linear elasticity.

Loredana Lanzani: Title: Applications of the Calderon-Zygmund theory to holomorphic singular integrals

Abstract: Let D be a domain in complex Euclidean space $\mathbb C^n$ with the complex dimension n being at least 2, so that the tool of conformal mapping is unavailable. The classical methods for the Cauchy-Szego projection, $\mathcal S_D$, associated to such D rely on apriori estimates of the Cauchy-Szego kernel and are most effective for smooth D (that is, D of class C^∞). In 1978 Norberto Kerzman and Eli Stein pioneered a new approach based on an operator identity that relates $\mathcal S_D$ to a holomorphic, Cauchy-type singular integral operator whose kernel was constructed in 1969 independently by G. Khenkin and E. Ramirez de Arellano.

The Kerzman-Stein identity is the earliest evidence pointing to a deep connection between several complex variables and the Calderon-Zygmund theory. (Incidentally, Calderon's iconic result on the Cauchy transform for a Lipschitz planar curve also appeared in 1978.) As the Calderon-Zygmund theory was further developed -- notably, through the $T(1)$ theorem -- its connection with several complex variables also evolved and ultimately led to the resolution, in

2017, of the L^p-regularity problem for $\mathcal S_D$ associated to strongly pseudoconvex D with minimal boundary regularity (namely, the class C^2), along with the resolution of the analogous problem for the Bergman projection $\mathcal B_D$.

In this talk I will highlight Eli Stein's central role in the development of this theory. I will also present new developments, by several authors, that have stemmed from Eli Stein's work, specifically: the weighted L^p-regularity of $\mathcal S_D$ and $\mathcal B_D$, and characterizations of (unweighted and weighted) L^p-regularity and L^p-compactness of the commutators $[b, \mathcal S_D]$ and $[b, \mathcal B_D]$ for strongly pseudoconvex D with minimal smoothness. I will close with open problems.

Akos Magyar: Title: Waring problem, nilpotent groups and averages along polynomial sequences

Abstract: We discuss some results on maximal and singular averages along polynomial sequences in discrete nilpotent groups, as well as related pointwise ergodic theorems for non-commuting transformations. These may be viewed as discrete analogues of averages over curves and surfaces in nilpotent Lie groups and some tools developed in the Euclidean case, such as almost orthogonality arguments, play an important role.

We will emphasize however the arithmetic nature of these problems, in particular a version of the Weyl inequality, the Waring problem and an extension of the classical circle method of exponential sums to the nilpotent settings. This is joint work with A. Ionescu, M.A. Mirek and T. Szarek.

Mariusz Mirek: Title: On recent developments in pointwise ergodic theory and harmonic analysis — my collaboration with Eli Stein.

Abstract: I will talk about my collaboration with Eli Stein in discrete harmonic analysis that recently led to some progress on the Furstenberg-Bergelson-Leibman conjecture.

Alex Nagel: Title: Geometric estimates for Bergman kernels on tube domains

Abstract: It is usually difficult to explicitly compute the Bergman kernel of a domain of holomorphy unless the domain has a large group of symmetries. Tube domains over a general convex base are an interesting sub-class which have only translational symmetry. We shall describe estimates for the Bergman kernel of such domains in terms of geometric properties of the base convex set. We obtain sharp diagonal estimates in the general case, and off-diagonal estimates in a number of cases including tubes over convex cones. Despite the lack of symmetry of the base we show that it is possible to use a kind of normalization and homogeneity to obtain the estimates. This is joint work with Malabika Pramanik.

Duong Phong: Title: Symplectic geometric flows and almost-complex structures

Abstract: The equations of unified string theories have led to several new interesting geometric flows. One of which is the Type IIA flow, which is a weakly parabolic flow on 6d symplectic manifolds whose underlying geometry turns out to be SU(3) holonomy, but for a *projected* Levi-Civita connection with respect to an almost-complex structure. We discuss this flow and many open related flows in symplectic geometry. This is joint work with Teng Fei, Sebastien Picard, and Xiangwen Zhang.

Jill Pipher: Title: Twenty years of boundary value problems: Stein's legacy in the theory of H^p spaces

Abstract: The advances in understanding boundary value problems for linear elliptic and parabolic equations in non-smooth domains and with non- smooth coefficients in the past twenty or so years have required and generated new ideas in harmonic analysis, geometric measure theory, PDE and potential theory. These advances involve second order elliptic/ parabolic equations associated to real matrices that are not symmetric or to complex matrices, to equations of higher order, in domains that are less regular than Lipschitz (chord-arc, uniform) and those with lower co-dimensional boundaries. In this talk, I will explain some of the fundamental connections between the advances in this theory with the ideas and techniques stemming from Stein's work on H^p spaces, notably the joint work with C. Fefferman.

Fulvio Ricci: Title: Multi-parameter Littlewood-Paley decompositions generated by families of dilations

Abstract: The topic of this talk is joint work with A. Hejna and A. Nagel. Its origin is in work from 1982 by D. Phong and E.M. Stein, where they discuss properties of compositions of two Calderon-Zygmund operators relative to two different types of dilations, one isotropic, the other parabolic.

A systematic treatment of the resulting class of operators, with extensions to any number of dilations, is contained in the AMS Memoirs paper of 2018 by A. Nagel, E.M. Stein, S. Wainger and myself. I will present the different kinds of Littlewood-Paley square functions that arise in this context and discuss their equivalence in defining an adapted h^1 space.

Chris Sogge: Title: Curvature and harmonic analysis on compact manifolds

Abstract: We shall explore the role that curvature plays in harmonic analysis on compact manifolds. We shall focus on spectral projection estimates and Strichartz estimates for solutions of the Schrodinger equation. We focus on gains that arise when the sectional curvatures of the manifold are negative.

Brian Street: Title: Maximal subellipticity

Abstract: The theory of elliptic PDE stands apart from many other areas of PDE because sharp results are known for very general linear and fully nonlinear elliptic PDE. Many of the classical techniques from harmonic analysis were first developed to prove these sharp results; and the study of elliptic PDE leans heavily on the Fourier transform and Riemannian geometry.

Starting with work of Hörmander, Kohn, Folland, Stein, and Rothschild in the 60s and 70s, a far-reaching generalization of ellipticity was introduced: now known as maximal subellipticity or maximal hypoellipticity. In the intervening years, many authors have adapted results from elliptic PDE to various special cases of maximally subelliptic PDE.

Where elliptic operators are connected to Riemannian geometry, maximally subelliptic operators are connected to sub-Riemannian geometry. The Fourier transform is no longer a central tool but can be replaced with more modern tools from harmonic analysis. In this talk, we present the sharp regularity theory of general linear and fully nonlinear maximally subelliptic PDEs.