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VALUES FOR GAMES WITHOUT SIDEPAYMENTS: SOME DIFFICULTIES WITH CURRENT CONCEPTS[1]

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Two solution concepts for games without sidepayments are considered: the stable bargaining solution proposed by Harsanyi [**6, 7**], and the λ-transfer value first proposed by Shapley [**19**]. Some examples of games are considered for which both solution concepts yield results which are highly counter-intuitive, and which seem to be inconsistent with the hypothesis that the games are played by rational players.

1. INTRODUCTION

THE GAME THEORY LITERATURE currently contains two well-developed solution concepts which can be applied to a cooperative game without side-payments to select an outcome of the game as its "value." The first of these is due to Harsanyi [**6, 7**], and is developed as a generalization of Nash's [**12**] solution of the two-person bargaining problem. Harsanyi's procedure selects a "stable bargaining solution" which is interpreted to be the outcome which would be selected by rational (utility-maximizing) players who are aware of all of the possibilities in the game.

The second solution concept, called the "λ-transfer value," was first proposed by Shapley [**19**] as an extension of the Shapley value [**18**] for games with sidepayments. Shapley [**19**, p. 260] writes that the idea was first motivated as an attempt to approximate Harsanyi's bargaining solution in the context of market games with many players, and was then perceived to have virtues of its own. The λ-transfer value has subsequently been studied and developed, primarily in the context of market games, by Aumann [**1**] and others (e.g., Aumann and Kurz [**2, 3**]; Champsaur [**5**]; Hart [**8**]; Mas-Colell [**9, 10, 11**]; Neymann [**13**]). However, like Harsanyi's stable bargaining solution, the λ-transfer value is defined for essentially all cooperative games, and it is customarily justified and interpreted without reference to markets. Since the Shapley value for games with sidepayments can be interpreted either as a stable outcome of bargaining (e.g., Harsanyi [**7**, Chapter 11]) or as an expected outcome or expected utility of playing a game (e.g., Shapley [**18**]; Roth [**15, 16**]), it can be argued that the λ-transfer value should be interpreted in either way.

Regardless of which interpretation is used, however, both the λ-transfer value and the stable bargaining solution can yield predictions which are highly counter-intuitive. In particular, Section 2 exhibits some games for which it seems that neither solution concept yields a result which is consistent with the hypothesis that

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the players of the game are rational utility-maximizers who are aware of all of the possible outcomes of the game.[2]

Section 2 contains the examples and an analysis of them; Section 3 contains a brief description of the stable bargaining solution, and applies it to the examples; Section 4 briefly describes the λ-transfer value and applies it to the examples. Section 5 contains a discussion of these results. Readers who are not interested in how the stable bargaining solution and λ-transfer value are computed may wish to read Section 2 and then skip directly to Section 5.

2. THE EXAMPLES

Consider a class of 3-player games defined by a single parameter p which varies between 0 and 1/2. For a given $p \in [0, 1/2]$, let $G(p)$ be the game in which the players acting alone can assure themselves of achieving a utility of 0, players 1 and 2 acting together can achieve the outcome (1/2, 1/2, 0), players 1 and 3 acting together can achieve the outcome $(p, 0, 1-p)$, and players 2 and 3 together can achieve $(0, p, 1-p)$. All three players acting together can achieve any convex combination of the vectors (1/2, 1/2, 0), $(p, 0, 1-p)$, and $(0, p, 1-p)$. Assume that no sidepayments of any sort are feasible between players.

This is essentially a game in characteristic function form,[3] so it can be represented by the set $N = \{1, 2, 3\}$ of players, the set of feasible outcomes $H(p)$ equal to the convex hull of the points $\{(1/2, 1/2, 0), (p, 0, 1-p), (0, p, 1-p), (0, 0, 0)\}$, and the characteristic function[4] V_p such that

$$V_p(i) = \{(u_1, u_2, u_3) | u_i \leq 0\} \qquad \text{for } i \in N,$$

$$V_p(\{12\}) = \{(u_1, u_2, u_3) | (u_1, u_2) \leq (1/2, 1/2)\},$$

$$V_p(\{13\}) = \{(u_1, u_2, u_3) | (u_1, u_3) \leq (p, 1-p)\},$$

$$V_p(\{23\}) = \{(u_1, u_2, u_3) | (u_2, u_3) \leq (p, 1-p)\}, \qquad \text{and}$$

$$V_p(N) = \{u = (u_1, u_2, u_3) | u \leq y \text{ for some } y \text{ in the convex hull of } \{(1/2, 1/2, 0), (p, 0, 1-p), (0, p, 1-p)\}\}.$$

I claim that, for $p < 1/2$, the payoff vector (1/2, 1/2, 0) is the unique outcome of the game consistent with the hypothesis that the players are rational utility

[2] Since I wish to show that these solution concepts can yield counter-intuitive results, the discussion will depend, to some extent, on informal, intuitive reasoning about the appropriate *interpretation* of mathematical ideas. Although I have tried to make the arguments as compelling as possible, I recognize that this sort of discussion may leave room for disagreement.

[3] That is, the payoffs available to a coalition are independent of the actions of the complementary coalition.

[4] In writing the characteristic function I have adopted the usual convention (cf. Aumann and Peleg [**4**]) that if x is in $V(S)$ and $y \leq x$, then y is also in $V(S)$. In general, however, y need not be a feasible outcome of the game, and in the game $G(p)$, only outcomes in the set $H(p)$ are feasible. Thus the game $G(p)$ is formally described by the triple $(N, V_p, H(p))$. This description is included only to insure that there is no ambiguity in the definition of the game $G(p)$, and readers who are unfamiliar with the characteristic function can safely ignore it.

maximizers. This is because, when $p < 1/2$, the outcome $(1/2, 1/2, 0)$ is *strictly* preferred by *both* players 1 and 2 to *every* other feasible outcome, and because the rules of the game permit players 1 and 2 to achieve this outcome without the cooperation of player 3. So in the game $G(p)$, for $p < 1/2$, there is really no conflict between players 1 and 2: their interests coincide in the choice of the outcome $(1/2, 1/2, 0)$, and the rules permit them to achieve this outcome.

This is perhaps clearest when $p = 0$, since in the game $G(0)$ players 1 and 2 have identical interests over the entire set of feasible outcomes. There is no pair of outcomes such that player 1 would choose one over the other, and player 2 would make the opposite choice. Furthermore, as far as players 1 and 2 are concerned, player 3 has nothing to offer in the game $G(0)$—his cooperation never offers either of them an increased reward. So, from the point of view of players 1 and 2, this game offers no prospect of reward different from the two-player game in which players 1 and 2 can achieve $(1/2, 1/2)$ if they agree, and $(0, 0)$ otherwise. The only rational outcome of this game, and consequently of the original game, is that players 1 and 2 should each receive a utility of $1/2$.

For $p < 1/2$, the outcome $(1/2, 1/2, 0)$ is not only the unique point in the core of the game $G(p)$, but is also the unique von Neumann-Morgenstern solution of the game.[5] It should be emphasized, however, that it is not this observation which prompts the conclusion that $(1/2, 1/2, 0)$ is the unique rational outcome of the game; the conclusion is due to the fact that players 1 and 2 agree on that outcome as preferable to all others.[6] (Note that this can never occur in a game with sidepayments, or in any game in which it is possible to freely redistribute wealth between players.)

Finally, note that the conclusions reached in this section apply only when $p < 1/2$. When $p = 1/2$ the game $G(p)$ is completely symmetric with respect to the players, so it is no longer the case that cooperation with player 3 offers strictly less to players 1 or 2 than cooperation with one another.

3. HARSANYI'S STABLE BARGAINING SOLUTION

Let G be a game with a set $N = \{1, \ldots, n\}$ of players and a set H of feasible, individually rational payoffs. Harsanyi's model can be divided into two parts. The

[5] By the core and solution of the game $G(p)$, I mean the core and solution of the feasible set $H(p)$ under the domination relation induced by the characteristic function V_p (cf. Aumann and Peleg [**4**]).

[6] Games without sidepayments are often described just by a set N of players and a characteristic function V. The set of feasible outcomes is then assumed to be equal to the set $V(N)$. Given the construction of the characteristic function (cf. footnote 4) this amounts to making the somewhat restrictive assumption that it is feasible to freely dispose of utility. Nevertheless, such an assumption would make no consequential difference in the analysis of the games $G(p)$ for $p < 1/2$. The principal effect of replacing the feasible set $H(p)$ by the set $V_p(N)$ would be that player 1, say, could achieve his maximum utility not only at the outcome $(1/2, 1/2, 0)$, but also at outcomes of the form $(1/2, q, 0)$ where $q < 1/2$, obtained by having player 2 dispose of some of the utility he could have obtained at the outcome $(1/2, 1/2, 0)$. Of course player 2 would prefer not to dispose of utility, and player 1 has no incentive to seek such a disposal of utility. (The hypothesis that the players are utility maximizers means player 1 doesn't want player 2 to dispose of utility in this way, since player 1 can't *want* anything which isn't captured by his utility function, and his utility remains constant at $1/2$.) Since each of players 1 and 2 can achieve his maximum utility only with the cooperation of the other, it is now straightforward to argue, as before, that $(1/2, 1/2, 0)$ must be the outcome of the game.

first part identifies one or more *bargaining solutions* of the game. In the event that more than one such bargaining solution is identified, the second part identifies a unique *stable bargaining solution.*[7]

A bargaining solution defined by the first part of the model consists of a feasible outcome u associated with a non-negative vector λ such that $\lambda \cdot u \geqslant \lambda \cdot x$ for any feasible outcome x. (That is, λ is the vector of coefficients of a hyperplane tangent to H at the point u.) To qualify as a bargaining solution, the payoff u_i to each player i must be the sum of dividends w_i^S which he receives from each coalition S of which he is a member, according to the rule that each coalition pays the maximum dividends that it can afford, subject to the restriction that $\lambda_i w_i^S = \lambda_j w_j^S$ for all $i, j \in S$. (The dividends w_i^S may be either positive or negative.)[8]

Let B be the (non-empty) set of bargaining solutions identified above. If B contains a unique element, then that is defined to be the unique stable bargaining solution of the game. Otherwise the unique stable bargaining solution is defined to be Nash's [**12**] solution of the pure bargaining game with outcome set H and disagreement point d such that $d_i = \min_{u \in B} u_i$. Thus the unique *stable bargaining solution*[9] is defined to be the outcome $u \in H$ such that $u \geqslant d$ and $\Pi_{i \in N} (u_i - d_i) \geqslant \Pi_{i \in N} (x_i - d_i)$ for all $x \in H$ such that $x \geqslant d$.

To see how this model applies to the games $G(p)$, first note that the plane $x_1 + x_2 + x_3 = 1$ is tangent to the set $H(p)$. When $\lambda = (1, 1, 1)$, the dividends which each coalition pays to determine a bargaining solution are $w_i^{\{i\}} = 0$ for $i \in N$, $w_1^{\{12\}} = w_2^{\{12\}} = 1/2$, $w_1^{\{13\}} = w_3^{\{13\}} = p$, $w_2^{\{23\}} = w_3^{\{23\}} = p$, and $w_1^N = w_2^N = w_3^N = -4p/3$. (Note that the grand coalition N must give a negative dividend, to preserve feasibility.) Summing these dividends, we find the bargaining solution for the game $G(p)$ corresponding to the weights $\lambda = (1, 1, 1)$ is the outcome $u = (1/2 - p/3, 1/2 - p/3, 2p/3)$.

It is easy to verify, however, that this is not the only bargaining solution of the game $G(p)$. In fact, each of the three extreme points of the set of Pareto optimal outcomes is the bargaining solution corresponding to some set of weights, for any game $G(p)$ with $p \geqslant 1/4$.

If a and c are positive numbers such that $pa = (1-p)c$, then the vector of weights $\lambda = (a, 0, c)$ yields the outcome $(p, 0, 1-p)$ as a bargaining solution of the game $G(p)$, since for these weights, the dividends $w_1^{\{13\}} = p$, $w_3^{\{13\}} = 1-p$, and all other dividends equal zero.[10] Similarly, if b and c are positive numbers such that $pb = (1-p)c$, then the bargaining solution corresponding to the weights $\lambda =$

[7] Only a brief description of the model will be given in this section. For a complete description, see Harsanyi [**7**, Chapter 12].

[8] If $\lambda_i = 0$ then w_i^S is not uniquely determined, and may be taken to be any quantity which yields a feasible solution (cf. Harsanyi [**7**, p. 258]). Of course, not every vector of weights yields dividends which produce a bargaining solution, but Harsanyi has shown that at least one such solution always exists for a wide class of games.

[9] Note that the stable bargaining solution need not be a bargaining solution.

[10] The requirement that $p \geqslant 1/4$ is needed in order that the plane defined by the equation $ax_1 + cx_3 = ap + c(1-p)$, which passes through the point $(p, 0, 1-p)$, be tangent to the set of feasible outcomes $H(p)$.

$(0, b, c)$ is the outcome $(0, p, 1-p)$, and the bargaining solution corresponding to $\lambda = (1, 1, 0)$ is the outcome $(1/2, 1/2, 0)$. Thus, for each player in the game, there is a bargaining solution at which he receives his minimum individually rational payoff.

Because there are multiple bargaining solutions, the second part of the model identifies a unique stable bargaining solution as follows.

PROPOSITION 3.1: *The stable bargaining solution for the game* $G(p)$ *with* $p \in [1/4, 1/2]$ *is the outcome* $u = (1/3, 1/3, 1/3)$.

PROOF: Since there is some bargaining solution at which each player receives 0, his minimum individually rational payoff, the disagreement point used to determine the stable bargaining solution is $d = (0, 0, 0)$. Thus the stable bargaining solution is the Nash solution of the three-person bargaining game with feasible set $H(p)$ and disagreement point d.

But the point $u = (1/3, 1/3, 1/3)$, which maximizes the product $\Pi_{i \in N} x_i$ over the simplex $\{x \geqslant 0 | \Sigma\, x_i = 1\}$ is always feasible in the game $G(p)$ for $p \in [1/4, 1/2]$; i.e., $(1/3, 1/3, 1/3) \in H(p)$ for all $p \in [1/4, 1/2]$. Consequently u is also the Nash solution to the bargaining game with feasible set $H(p)$, so u is the unique stable bargaining solution of the game $G(p)$.

4. THE λ-TRANSFER VALUE FOR THE GAMES $G(p)$

Let $\lambda = (a, b, c)$ be a non-negative vector of weights, at least one of which is not zero. Define the *weighted game* $G_\lambda(p)$ to be the game in which each coalition S can achieve the payoffs (au_1, bu_2, cu_3), where (u_1, u_2, u_3) is a payoff which that coalition could achieve in the game $G(p)$. Define the weighted game with sidepayments $g_\lambda(p)$ to be the game in which a coalition can achieve any distribution of utility whose sum over members of the coalition does not exceed the sum of the utilities available to that coalition at some outcome in the game $G_\lambda(p)$. Since the games $G_\lambda(p)$ are in characteristic function form, so are the games $g_\lambda(p)$, which can be represented by the characteristic function $v_{\lambda p}$ given by

$$v_{\lambda p}(i) = 0 \qquad \text{for } i = 1, 2, 3;$$

$$v_{\lambda p}(12) = \frac{a+b}{2};$$

$$v_{\lambda p}(13) = pa + (1-p)c;$$

$$v_{\lambda p}(23) = pb + (1-p)c;$$

$$v_{\lambda p}(123) = \max\left\{\frac{a+b}{2}, pa + (1-p)c, pb + (1-p)c\right\}.$$

Consequently, the Shapley value $\phi(v_{\lambda p})$ is always well defined for the game $g_\lambda(p)$. In general, the Shapley value for the game $g_\lambda(p)$ need not be feasible for the game $G_\lambda(p)$. However if no utility transfers are required to achieve the Shapley value for the game $g_\lambda(p)$, then it is a feasible outcome of the game $G_\lambda(p)$, and in this case it is called a λ-transfer value for the game $G(p)$. Formally, a *λ-transfer value* for the game $G(p)$ is a feasible outcome $u=(u_1, u_2, u_3)$ of the game $G(p)$ such that $(au_1, bu_2, cu_3)=\phi(v_{\lambda p})$ for some vector of weights $\lambda=(a, b, c)$.

Although there are a number of ways to motivate this definition, the argument which has been most influential is made in two parts.[11] The first part considers the original game (i.e., the game $G(p)=G_\lambda(p)$ for $\lambda=(1, 1, 1)$) and the corresponding game with sidepayments. If the Shapley value of the sidepayment game is feasible in the original game, then it is justified as the value for the original game by invoking Nash's [**12**] principle of "independence of irrelevant alternatives." Aumann [**1**, Section 6][12] puts the argument in a picturesque way by supposing that the players first negotiate under the assumption that utility is transferable, and only find that this is not the case after reaching an agreement. If the agreement requires no utility transfers to be made, however, then it could still be implemented.

Since this procedure may not yield a feasible outcome, the second part of the argument notes that, for any *strictly positive* vector of weights λ, the game $G_\lambda(p)$ is equivalent to $G(p)$ in the sense that the utility functions of the players are unique only up to positive linear transformations. So if some such λ can be found which yields a feasible outcome, the argument of the previous paragraph should still apply.[13]

In order to prove an existence theorem for a broad class of games (Shapley [**19**]) it is necessary to extend the definition to include vectors of weights λ which may be zero in some components. However the equivalence argument breaks down in this case, and this extension is customarily viewed as simply a technical expedient. In many cases of interest, existence can be obtained without resorting to weights of zero.

We are now in a position to study the λ-transfer value for the game $G(p)$.

PROPOSITION 4.1: *For any game $G(p)$ with $p\in[0, 1/2]$, the outcome $u=(1/3, 1/3, 1/3)$ is a λ-transfer value for $\lambda=(1, 1, 1)$.*

PROOF: Observe that when $\lambda=(a, b, c)=(1, 1, 1)$, then $v_{\lambda p}$ is independent of p and is the completely symmetric characteristic function representing 3-person majority-rule. Consequently, $\phi(v_{\lambda p})=u=(1/3, 1/3, 1/3)$ for all p. Furthermore, the outcome $u=(1/3, 1/3, 1/3)$ is feasible in the game $G(p)$ for all $p\in[0, 1/2]$, since it lies on the line joining the outcomes $(1/2, 1/2, 0)$ and $(p/2, p/2, 1-p)$. Consequently u is the λ-transfer value for the game $G(p)$.

[11] See Shapley [**19**] and Aumann [**1**] for a more complete account.

[12] Aumann is actually considering a modified version of the value described here.

[13] The vector λ can be interpreted as a vector of exchange rates according to which utility can be transferred in the sidepayment game—hence the name "λ-transfer value."

Of course, another vector λ of weights could yield another λ-transfer value. For instance, it is not difficult to verify that the weights $\lambda = (a, b, c) = (1, 1, 0)$ yield the λ-transfer value $u = (1/2, 1/2, 0)$ for the game $G(0)$. In view of the analysis of the game $G(p)$ given in Section 2, we wish to know in exactly what circumstances weights λ can be found such that the λ-transfer value gives player 3 a payoff of zero in games $G(p)$ for $p < 1/2$.

PROPOSITION 4.2: *If* $u = (u_1, u_2, u_3)$ *is a* λ*-transfer value for a game* $G(p)$ *with* $p \in (0, 1/2]$, *then* $u_3 > 0$.

PROPOSITION 4.3: *If* $u = (u_1, u_2, u_3)$ *is a* λ*-transfer value for the game* $G(p)$, *and if* $\lambda = (a, b, c)$ *with* $c > 0$, *then* $u_3 > 0$ *for all* $p \in [0, 1/2]$.

PROOF: When $p > 0$ at least one of $v_{\lambda p}(13)$ or $v_{\lambda p}(23)$ must be positive, since not all of the components of λ may be equal to zero. When $c > 0$, both of these quantities must be positive. Consequently, under the conditions of both propositions, the Shapley value for the game with sidepayments gives player 3 a positive payoff for any λ. Since a λ-transfer value corresponds to the Shapley value of some sidepayment game, the result follows.

Thus the outcome $(1/2, 1/2, 0)$ can be achieved as a λ-transfer value only in the game $G(0)$, and even then only by first multiplying the utility function of player 3 by zero.

5. DISCUSSION

In order to simplify the discussion, consider a unique λ-transfer value for the games $G(p)$ by taking $\lambda = (1, 1, 1)$. (By Propositions 4.2 and 4.3, most of the discusson will apply as well to any other choice of weights λ.)

Both the stable bargaining solution and the λ-transfer value select the outcome $u = (1/3, 1/3, 1/3)$ for any game $G(p)$ with $p \in [1/4, 1/2]$. (In fact the λ-transfer value selects the outcome u for any $p \in [0, 1/2]$.) In view of the analysis presented in Section 2 of the games $G(p)$ for $p < 1/2$, it no longer seems tenable to interpret either solution concept as yielding a "stable" outcome of the game.[14] Perhaps something more needs to be said, however, about the idea that the outcome $(1/3, 1/3, 1/3)$ might represent some sort of expected outcome or expected utility. These are two distinct ideas, and must be addressed separately.

To interpret $u = (1/3, 1/3, 1/3)$ as an expected outcome, it must be maintained either that u is the only outcome likely, in some sense, to occur, or else that there is a set of likely outcomes distributed in such a way as to make u their expectation. For the three-person majority game with sidepayments (or for the game $G(p)$ with $p = 1/2$), the complete symmetry of the game makes both of these positions defensible, since u is the only efficient outcome which is symmetric. For instance,

[14] In private correspondence, John Harsanyi has informed me that he is currently developing a solution concept which *does* yield the outcome $(1/2, 1/2, 0)$ as the solution to the games $G(p)$ with $p < 1/2$.

any theory which depends only on the structure of the game and which finds the outcome $(1/2, 1/2, 0)$ to be stable must also find the outcomes $(1/2, 0, 1/2)$ and $(0, 1/2, 1/2)$ to be stable, and u is the mean of these three outcomes.

But when $p > 1/2$, the game $G(p)$ is *not* symmetric (and in particular the outcomes $(1/2, 0, 1/2)$ and $(0, 1/2, 1/2)$ are not feasible). If u is to be interpreted as an expected outcome, then one must be prepared to argue that there are some "likely" outcomes which can be balanced against $(1/2, 1/2, 0)$ to yield u as an expected outcome. In view of the analysis presented in Section 2, this would seem to be a difficult positon to defend. Note that I am essentially suggesting that the principle of "independence of irrelevant alternatives" is inappropriate under this interpretation—the notion of an "expected outcome" seems to require consideration of feasible outcomes other than the one chosen for the solution.

Finally, consider whether the vector $u = (1/3, 1/3, 1/3)$ might reasonably represent the expected utility of playing one of the positions in a game $G(p)$. Of course, any function which assigns numerical values to the positions in a game can be used as a utility function in the limited sense that it induces binary choices which are transitive and complete (since the ordering of the real numbers has this property). To decide whether a function is a reasonable utility function, it is necessary to consider what kind of preferences it reflects. The Shapley value for games with sidepayments was studied in this context in Roth [**15, 16**].

In order for the λ-transfer value (with $\lambda = (1, 1, 1)$) to represent an individual's utility for playing in the games $G(p)$, it must be that he is indifferent between playing in any position of any game $G(p)$ for $p \in [0, 1/2]$. In particular, he must be indifferent between playing position 1 or position 3 in any game $G(p)$ for $p < 1/2$, and indifferent between playing position 3 in the game $G(p)$ or in the game $G(1/2)$, since all of these prospects have a λ-transfer value of $1/3$. In view of the preceding discussion, these preferences are inconsistent with the notion that a rational player's preferences over games should be influenced by the payoff he might reasonably expect to achieve in those games.

Thus, for a simple family of superadditive games without sidepayments, both the stable bargaining solution and the λ-transfer value yield results which are difficult to justify. Unfortunately, the analysis of these games does not itself suggest any alternative theory for general cooperative games, since the arguments depend critically on the extreme simplicity of the games $G(p)$, which permitted us to analyze the games as in Section 2, essentially from first principles.

What this analysis does suggest is that, at the very least, some modifications are required in the existing theory.[15]

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[15] For other suggestive examples, see Owen [**14**, Example 2] or Shafter [**17**].

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