On the Non-Transferable Utility Value: A Comment on the Roth-Shafer Examples

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Econometrica, Volume 53, Issue 3 (May, 1985), 667-678.

Econometrica, Vol. 53, No. 3 (May, 1985)

NOTES AND COMMENTS

ON THE NON-TRANSFERABLE UTILITY VALUE: A COMMENT ON THE ROTH-SHAFER EXAMPLES[1]

BY ROBERT J. AUMANN

1. INTRODUCTION

IN GAME THEORY, the term "solution concept" denotes a correspondence between games and outcomes (or sets of outcomes). Two familiar examples of solution concepts are the Nash Equilibrium Point and the Core. Both are successful tools of economic analysis: applied to a variety of contexts, they yield important and interesting results. The Core is particularly successful in classical[2] market contexts.

Another solution concept is Shapley's value [**47**]; more generally,[3] his Non-transferable-Utility (NTU) value[4] [**49**]. While not as well-known as the core, it is in some ways even more "successful": it has been applied to a broader variety of contexts, often yielding interesting results.[5]

Several years ago, A. Roth [**43**] constructed a class of examples in which, he argued, the NTU value looks strange and counterintuitive; specifically, in which there are very strong, compelling reasons leading to an alternative outcome, not consistent with the NTU value. He concluded that "at the very least, some modifications are required in the existing theory." While far from universally accepted in the profession, Roth's work has had a considerable echo.

The main purpose of this paper is to rebut Roth's. We make two points:

(i) Roth's reasoning is unsound; specifically, the arguments for the alternative outcome are not nearly as compelling as they appear at first.

(ii) Even if the arguments were sound, the examples would by no means justify abandoning the NTU value as an analytic tool, or even modifying it. A solution concept is not a theorem, and one counterintuitive example is not sufficient to make us abandon an otherwise successful tool. Most popular solution concepts are beset by counterintuitive examples; we will adduce just two, one for the Nash Equilibrium Point, and one for the Core.

[1] This work was supported by National Science Foundation Grant SES 80-06654 at the Institute for Mathematical Studies in the Social Sciences, Stanford University, and by the Institute for Advanced Studies at the Hebrew University of Jerusalem. We gratefully acknowledge conversations and correspondence with K. Arrow, K. Binmore, M. Kurz, M. Maschler, A. Roth, and L. Shapley. Of course they are not responsible for the views expressed here. Roth, indeed, will speak for himself: He plans to publish a response

[2] I.e., without political elements, public goods, taxation, increasing returns, fixed prices, and so on.

[3] As defined in [**47**], the value applies to transferable utility (TU) games only. NTU games generalize TU games; every TU game is an instance of an NTU game, but not conversely. The NTU value, when applied to coalitional[24] TU games, coincides with the TU value.

[4] Sometimes called the *λ-transfer* value, an unfortunate practice, as it involves no transfers. Shapley [**49**] used the term "λ-transfer value" in a different sense, closer to the plain meaning of the words.

[5] Until the last decade, most of the applications of the value concept were in a TU environment. The literature on applications of the TU value is far too voluminous to be catalogued here. The last decade has seen more and more applications of the NTU value in the "strict" sense, i.e., to environments that are not TU. These include classical markets [**1, 10, 11, 12, 20, 24, 28, 46, 50**], production with increasing average returns [**29**], taxation [**4, 5, 14, 15, 26, 27, 36, 37**], public goods [**6, 7, 40, 41**], monopoly [**16**], rationing and fixed prices [**3, 15**], incomplete information [**30**], and general theories of justice [**8, 9, 49, 51**] (to which the core is inapplicable because, inter alia, it is often empty). Note also that the early works of Nash [**31, 33**] on bargaining and threatening in a two-person context are in fact applications of the "strict" NTU value. (For axiomatizations of the NTU value and additional treatments of a general nature, see [**2, 21, 22, 23, 24, 25, 35, 38, 39, 42**].)

Point (ii) is presented in Section 2. Most of the remaining sections are devoted to our main thesis, Point (i); readers in a hurry may confine themselves to Section 3, which presents the gist of the argument informally.

Back-to-back with [**43**], W. Shafer [**46**] published a different set of examples meant to show that the NTU value may yield counterintuitive results. While similar in principle to Roth's, these examples are set in a special context that makes them in some ways more compelling. Nevertheless, they fit well into the general framework of the NTU value; this will be discussed in Section 8.

This paper focuses on the reasonableness or unreasonableness of various *outcomes* of the Roth and Shafer games; it is not concerned with the internal workings of the NTU value. Therefore we do not find it necessary to quote the definition of the NTU value, which may be found in almost any paper on the subject. Conceptual discussion of the definition as such may be found, e.g., in [**1, 49**].

2. COUNTERINTUITIVE EXAMPLES FOR OTHER SOLUTION CONCEPTS

Consider first the Nash Equilibrium Point (EP), the game theoretic concept that is perhaps best-known and most frequently applied in economics. There are very simple, natural non-zero-sum two-person games that have a unique EP $\sigma = (\sigma_1, \sigma_2)$, which yields each player only his security level (i.e., his maxmin value, the amount he can guarantee for himself), but such that σ_i does not, in fact, *guarantee* the security level. For example, the game[6] in Figure 1 has a unique EP, consisting of (1/2, 1/2) for each player, and the expected outcome is (3, 3). But in using those strategies, each player runs the risk of receiving less than 3 if the other should play his second strategy. This risk is quite unnecessary, since player 1 has the maxmin strategy (3/4, 1/4) available, which assures him of 3 regardless; similarly player 2 has strategy (1/4, 3/4). Under these circumstances, it is hard to see why the players would use their equilibrium strategies.[7]

Next, we turn to the Core, also widely applied in economics. Consider a market in totally complementary goods, e.g., right and left gloves. There are four agents. Initially 1 and 2 hold one and two left gloves respectively, 3 and 4 hold one right glove each. (In coalitional form, $v(1234) = v(234) = 2$, $v(ij) = v(12j) = v(134) = 1$, $v(S) = 0$ otherwise, where $i = 1, 2$ and $j = 3, 4$.) The Core has a unique point, namely (0, 0, 1, 1); that is, the owners of the left gloves must simply give their merchandise, for nothing, to the owners of the right gloves. This in itself seems strange enough. It becomes even stranger when one realizes that Agent 2, simply by throwing away one glove—an action that he can perform by himself, without consulting anybody—can make the situation completely symmetric (as between 1, 2 and 3, 4). Appeals to "competition" ring hollow. With such small numbers—two traders on each side—the market can hardly be deemed competitive;

2,6	4,2
6,0	0,4

FIGURE 1

[6] Examples of this kind have been in the folklore of game theory for a long time. For a discussion, see, e.g., [**18**, p. 125].

[7] The equilibrium and maxmin strategies are mixed, but that is not an issue; if one excludes mixed strategies, one can still construct an example in which these phenomena occur, simply by explicitly adding rows and columns to the original game that contain the payoffs of the appropriate mixed strategies.

certainly not here, where a single agent can, by his own actions, improve the situation so dramatically for himself.[8]

Do these examples imply that we should abandon or modify the EP or the Core? We think not. At some point, we should ask ourselves how such counter-intuitive examples fit into the conceptual framework of game theory, and of theory in the social sciences in general. But not in this article. Here we wished only to show that at worst, the Roth example puts the NTU value into a class with the EP and the Core; and in the sequel, we mean to show that it doesn't even do that.

3. THE ROTH EXAMPLE

Let p be a parameter with[9] $0 < p < 1/2$. There are three players, who must share 1. By himself, each player can get 0. If Players 1 and 3, or 2 and 3, form a coalition, then 3 gets a utility of $1-p$ (the larger amount), and the other player gets p. If 1 and 2 form a coalition, they get 1/2 each. If all three form a coalition, they may use a random device of their choosing to pick a 2-person coalition, which must then divide as above. No other outcomes are possible.

The unique NTU value of this game is (1/3, 1/3, 1/3). But Roth argues that 3 is weak, because he can only offer 1 and 2 a payoff of p, which is $<1/2$. Players 1 and 2 would therefore spurn 3's offers, and gravitate toward each other. Roth concludes that the outcome *must* be (1/2, 1/2, 0); it is the "unique outcome ... consistent with the hypothesis that the players are rational utility maximizers ... the outcome (1/2, 1/2, 0) is *strictly* preferred by *both* players 1 and 2 to *every* other feasible outcome ... So ... there is really no conflict between players 1 and 2: their interests coincide in the choice of the outcome (1/2, 1/2, 0), and the rules permit them to achieve this outcome" [**43**, pp. 468-9; his emphasis].

At first, this reasoning sounds compelling. But let's look a little closer. Suppose the players and the rules have just been announced on television. The amount 1 to be shared may be fairly large, so the players are rather excited. Suddenly the phone rings in 1's home; 3 is on the line with an offer. At first 1 is tempted to dismiss it. But then he realizes that if he does so, and if 3 manages to get in touch with 2 before he (1) does, then he won't get anything at all out of the game, unless 2 also rejects 3's offer. "But wait a minute," 1 now says to himself; "2 will only reject 3's offer if he thinks that I will reject it. When he gets 3's phone call, he will go through the agonizing that I am going through now, and will realize that in his situation I would also agonize. We seem to be caught in a web of circular reasoning. It is rational for me to reject 3's offer only if it is rational for 2 to reject it, and this in turn depends on its being rational for me to reject it. In short, I should reject 3's offer only if it is pretty clear to start with that I should reject it. I'm beginning not to like this one bit."

At this point, 1 breaks into a cold sweat. "Are you still there?", he says anxiously into the receiver. "Yes," says 3, "but I'm getting a little impatient." 1 sighs with relief. "You have a deal," he says.

To illustrate the force of this reasoning, suppose the amount to be divided is \$100,000, and that $p = \$49{,}000$. When 1 gets 3's phone call, he must choose between (i) getting \$49,000 with certainty, on the spot; and (ii) getting \$50,000 if he is convinced that 2 is convinced that he (1) will reject 3's offer (or if he can get in touch with 2 before 3 does), and getting 0 otherwise. In my opinion there is little doubt that 1 would accept the \$49,000, even though the \$1,000 he foregoes is by no means a negligible sum.

[8] The archetype of this genre of examples is the market with one seller and two buyers [**34**, p. 610 ff.], in which the unique core point is (1, 0, 0). Cf. also [**13, 48**].

[9] The end-points have special properties requiring separate treament. When $p = 1/2$, the game is symmetric, and all agree that (1/3, 1/3, 1/3) is the appropriate "value." For a discussion of $p = 0$, see note 20.

In short, Roth's statements are simply incorrect. (1/2, 1/2, 0) is *not* the "unique outcome consistent with the hypothesis that the players are rational utility maximizers." Another outcome that may well be[10] consistent with this hypothesis is that resulting if any two players who meet immediately close a deal. Indeed, if each player thinks that the others will do this, then *to maximize his own utility*, he must do so as well. We are of course not asserting that rational utility maximization implies this as the unique outcome. But it is certainly *consistent* with rational utility maximization.

Where Roth went wrong is in ignoring the crucial distinction between a single rational decision maker, and several of them. If 1 and 2 had had a single "manager", his arguments would have been airtight. But one of the central questions of cooperative game theory has always been, which coalition will form? And to this question, Roth's arguments do not speak convincingly. It is true that each of 1 and 2 would have liked $\{1, 2\}$ to form. It is also true that *acting together*, they can bring this about. But acting separately, neither one of them can bring it about. And it will not come about without a kind of mutual reliance that has little to do with ordinary individual utility maximization, and that, because of its riskiness, may be totally unreasonable.

4. A FIFTY-PERSON GAME

To underscore the distinction between Roth's "rationality" and the ordinary kind, consider the following 50-person game: Three million dollars are to be divided. Each of Players 1 through 49 can form a two-person coalition with Player 50, which *must* split 59:1 (in favor of 50), yielding the "small" partner $50,000. The only other coalition that can get anything consists of all the players 1 through 49, which *must* split evenly, yielding each partner about $61,000. As before, the all-player coalition has the option of choosing a smaller coalition by a random device.

The full force of Roth's reasoning applies to this game; there is no reason that his kind of "rationality" should apply any the less to 49 people than to 2. So presumably, he would predict with certainty that Players 1 through 49 will form a coalition and split evenly; in the role of Player 1, he would reject any overtures from 50 with dignity but firmness. Perhaps we are irrational, but for his sake, we hope we are not Player 2; for he can be assured that we would accept any offers from 50 with alacrity, while he is out there trying to round up the other fellows.

5. SOME FORMAL BARGAINING MODELS

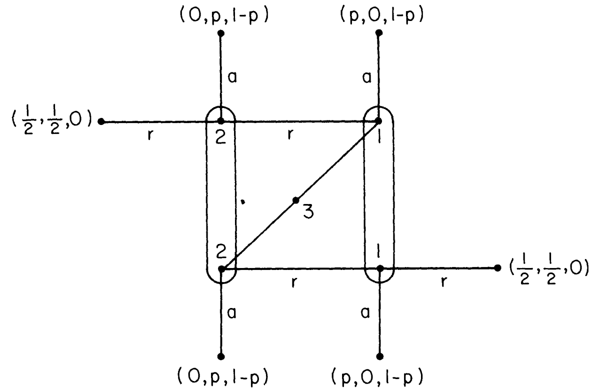
Let us return to the game of Section 3. In commenting on Roth's paper, J. Harsanyi [**19**] suggests that all cooperative solution notions be abandoned. To deal with cooperative games (such as the one before us), he suggests constructing formal bargaining models, and analyzing them as non-cooperative games. This program, which goes back to Nash [**32**], has some serious pitfalls, as we will see in Section 7. Nevertheless, it is useful as a touchstone for the informal kind of reasoning that characterizes both Roth's paper and the previous sections of this one; it clarifies and sharpens our thinking. Also, it enables us to apply the familiar formal concepts of non-cooperative game theory.[11]

One simple bargaining model is the following: A player i is picked at random and given the "initiative". That is, i chooses another player j, and makes him an offer. If j rejects the offer, i makes an offer to the remaining player k; but k does not know of the previous offer to j. If k also rejects i's offer, the coalition $\{j, k\}$ forms.

The interesting case is that in which 3 gets the initiative. This results in a subgame whose extensive and strategic forms are depicted in Figure 2. Its EP's include (r, r, μ)

[10] Formally, this may depend on p and on the bargaining procedure. Cf. Section 5.

[11] EP's and their variants.



3 chooses 1

	r	a
r	$\frac{1}{2}, \frac{1}{2}, 0$	$0, p, 1-p$
a	$p, 0, 1-p$	$p, 0, 1-p$

3 chooses 2

	r	a
r	$\frac{1}{2}, \frac{1}{2}, 0$	$0, p, 1-p$
a	$p, 0, 1-p$	$0, p, 1-p$

FIGURE 2.—r and a denote, respectively, "reject 3's offer" and "accept 3's offer."

and (a, a, μ), where μ is any mixed strategy of 3; denote these EP's by R and A respectively.

The phrase used by Roth—"outcome of the game consistent with the hypothesis that the players are rational utility maximizers"—is an excellent description of an EP. Indeed, an EP is *defined* as an outcome at which each player maximizes his utility, given that the others are at this outcome. Roth claims that such an outcome *must* lead to the coalition $\{1, 2\}$. Since this is not the case for the equilibrium point A, we conclude that at least in this bargaining model, Roth's assertion is incorrect.

While EP's are always consistent with rationality, we certainly do not claim that they are always consistent with reasonableness; witness the example in Section 2. But A does happen to be a rather reasonable specimen. First, it is "trembling hand" perfect in a very strong sense;[12] each player's action remains rational even if he is not entirely certain that all will play according to A. Second, for both 1 and 2 it is "strict," i.e., prescribes the *unique* best response to itself;[13] thus rationality not only allows 1 and 2 to accept an offer from 3, it *requires* them to do so (if each believes the other will accept). Of course R enjoys similar "reasonableness" properties.

When we compare A to R, we find that from the point of view of payoff, both 1 and 2 prefer R. But payoff is not the only consideration when choosing an EP. There is also

[12] The definition [**45**] of a perfect EP requires only that there *exist* perturbations of the game with EP's close to it. In our case, this is so for *all* perturbations.

[13] It is the lack of strictness that enables the pathology of the example in Section 2. Unfortunately, there are many important games that do not possess strict EP's. In our case, neither A nor R are strict for 3, but only because 3's choice never affects his payoff. Thus his choice is a matter of total indifference to him, so it is reasonable to assume that 1 and 2 perceive 3's strategy as mixed. (Harsanyi [**18**, p. 104] uses "strong" for what is here called "strict;" but "strong" has a different meaning in most of the literature.)

	r	a
r	$\frac{1}{2}, \frac{1}{2}, 0$	$\frac{1}{4}, \frac{1}{4}+\frac{1}{2}p, \frac{1}{2}(1-p)$
a	$\frac{1}{4}+\frac{1}{2}p, \frac{1}{4}, \frac{1}{2}(1-p)$	$\frac{1}{6}+\frac{1}{3}p, \frac{1}{6}+\frac{1}{3}p, \frac{2}{3}(1-p)$

FIGURE 3

the problem of coordination.[14] How sure can each player be that the others will play according to it? And what is the cost if they do not?

From this point of view, A is distinctly preferable. Indeed, A requires no coordination at all; at the moment that 1 or 2 accepts 3's offer, he is *assured* a payoff of p, no matter what strategy the other one uses. On the other hand, R requires very much coordination: If 1 or 2 rejects 3's offer, he will get 0 unless the other one also rejects.

Taking all this into account, which EP seems more likely? This depends on p. If p is close to $1/2$, the players will probably perceive the coordination and safety issues as paramount, and play a (i.e., accept 3's offer); if p is close to 0, they will probably forego safety for the lure of a higher payoff, i.e. play r. One can't set a precise boundary; moreover, considerations other than the size of p may enter, as we will see in Section 8. In any case, a blanket assertion that $(1/2, 1/2, 0)$ *must* be the outcome, no matter what p is, is totally unjustified.

Of course, if 1 or 2 get the initiative, then all perfect EP's do result in $(1/2, 1/2, 0)$, though ordinary EP's need not.

Another bargaining model is the following: The three pairs of players are ordered at random, and in this order, are given the opportunity to agree; the first pair that does so forms a coalition. If no agreement is reached, all players get 0. Each player remembers which proposals he has rejected, but is not informed of proposals not involving him.

In one EP of this game, all players reject all proposals. This sounds rather unreasonable, and indeed this EP is neither perfect nor strict (for any player). It seems reasonable to consider only strategies in which 1 and 2 agree if they meet, and 3 makes an offer to the first player he meets. Indeed this is so at all perfect EP's. The only remaining issue is whether 1 and 2 should reject (r) or accept (a) 3's offer if they meet him. Figure 3 depicts the strategic form.

The analysis of this model is qualitatively similar to that of the first model, though there are differences. When $p > 1/4$, both (r, r) (henceforth R) and (a, a) (henceforth A) are perfect and strict EP's, and the comparison of A to R is much like in the first model. Unlike in the first model, A is here no longer an EP when $p < 1/4$. On the other hand, here A always yields to 3 more than his NTU value of $1/3$, whereas in the first model it always yields him less.[15]

6. PRE-PLAY COMMUNICATION DOESN'T HELP

The analyses in the previous section *remain valid if pre-play communication is permitted.*[16] Suppose that before formal bargaining begins—i.e. before binding agreements can actually be made—there is a period during which players may converse and make tentative, non-binding agreements; moreover, all pairs of players actually get an opportunity to talk to each other. Suppose now that 1's thinking is such that in the absence of pre-play communication, he would play a; this is because his suspicion that 2 might also play a outweighs the prospect of the additional payoff otherwise. A pre-play conversation between 1 and 2 could alter 1's decision only if it could somehow allay his fear that 2 will play a.

[14] We use the term "coordination" for the problem of how players choose one EP from among a multiplicity of EP's. Harsanyi [**18**, p. 133] uses the same term in the related but narrower sense of choosing an EP from a multiplicity of EP's with the same payoff.

[15] One must remember to average into the payoff the possibility that 1 or 2 get the initiative.

[16] I.e., if the game is *vocal* [**18**, p. 112].

But if 2 really does intend to play *a*, he will want 1 to play *r*, since that improves 2's chances of getting an offer. To this end, 2 will be delighted to enter into a non-binding agreement with 1 to play *R*. Since 2 is motivated to make such an agreement no matter what he actually intends to play, such an agreement can give 1 no information. Similarly it can give 2 no information; the agreement is a dead letter as soon as it is made. The conversation between the players is therefore useless; in these games, it cannot help to resolve the coordination problem, and so cannot affect the outcome.

7. CONCLUSIONS FROM THE BARGAINING MODELS

One should neither overestimate nor underestimate the importance of the kind of formal bargaining model considered above. The quantitative results cannot be considered particularly significant. There are too many different possibilities for constructing a bargaining model, and the numerical results are too sensitive to its specific form. Moreover, "real" bargaining is too unstructured to be faithfully represented by such a model; it seems impossible adequately to model all the subtleties of communication, timing, information, etc., that are inherent in real multi-person bargaining. *Any* formal model will have serious artificialities, which will distort the numerical results.[17]

In principle, though, Nash's program sounds very attractive. Indeed, if one accepts individual utility maximization as *the* driving force of game theory and economics, it is almost inescapable. At least, this is so if one views cooperative games simply as games in which binding agreements can be made, without reading other connotations into the word "cooperative."

Fortunately, Nash's program can be used qualitatively, without committing oneself to a specific bargaining model. Roth's solution is a case in point. Roth asserts that the logic of the situation implies that the outcome must *always* be $(1/2, 1/2, 0)$. To refute this, it is in principle sufficient to point to one bargaining model in which it is not so, which we have done. Neither can $(1/2, 1/2, 0)$ be considered as an amalgam arising from different bargaining models; its extreme nature—obviously 3 can't get *less* than 0—implies that if $(1/2, 1/2, 0)$ is an average of outcomes, then it is *always* the outcome, and we are back to the previous argument.

There is another important qualitative use of Nash's program. Starting from analyses like those in Section 5, one asks which part of the reasoning depends critically on the specific bargaining model, and which is more generally valid. In our case, the numerical results depend critically on the specific bargaining model. But the reasoning according to which the coalitions $\{1, 3\}$ or $\{2, 3\}$ *can* form in equilibrium, and for large p are even likely to form, is of quite general validity.[18] Already in Section 3, where the setting was

[17] Another difficulty is that of multiple EP's; I have not stressed it because it is so well known. Non-uniqueness is of course ubiquitous in game theory as well as economics; but bargaining models are especially prone to having great swarms of EP's, which often render the analysis almost useless. Harsanyi and Selten [**17**] have developed several related methods, of considerable depth, for assigning unique EP's to games; but these methods are not nearly as persuasive as the more fundamental concepts of the noncooperative theory (EP's, perfect EP's, etc.).

[18] One must be careful not to overstate the case. When we say that the conclusion is "of quite general validity," we do not mean that it holds in every conceivable bargaining model. We do mean that it holds in a wide range of fairly natural contexts; it does not depend on specific structures. It may well be possible to construct bargaining models for which the conclusion is false. If, for example, we demand that all bargaining take place in public, and that the extensive form be finite, then all *perfect* EP's do lead to $(1/2, 1/2, 0)$, though ordinary EP's still need not. But this is a rather artificial context; the special circumstances enable one to work backwards from the end to obtain an anti-intuitive result, much like in the hangman's paradox or the finitely repeated prisoner's dilemma. One man's meat is another man's poison, of course; Roth may say that a bargaining procedure cannot be considered "natural" *unless* all bargaining takes place in public. In any case, we take no dogmatic stance, and make no assertion that any particular outcome is the only "rational" one.

completely amorphous, we argued that these are likely outcomes; what we are able to add now is that in fact, they correspond to perfect EP's of a bargaining game, though the setting remains amorphous. The reasoning according to which pre-play communication does not significantly change matters is also of quite general validity.

Let us state some conclusions. On the negative side, I think we have shown fairly conclusively that $(1/2, 1/2, 0)$ need *not* be the outcome. It is more difficult to reach definitive conclusions on the positive side. Given a sufficiently abstract,[19] symmetric situation, it does appear that the coalitions $\{1, 3\}$ and $\{2, 3\}$ are less likely to form than $\{1, 2\}$; this is because $\{1, 2\}$ will form as soon as its members have the opportunity to close a deal, which is not necessarily so for $\{1, 3\}$ and $\{2, 3\}$. Moreover, the smaller p is, the less likely it is that $\{1, 3\}$ or $\{2, 3\}$ will form.[20] But whereas 3's *chance* of getting into a coalition is smaller than that of the others, his *payoff* $1-p$ if he does get in is larger, and these two effects vary in opposite directions as p varies. All in all, perhaps $(1/3, 1/3, 1/3)$—the NTU value—does reflect the qualitative features of the situation quite well.

8. SOME SPECIAL CONTEXTS

We start with a political story. In many parliamentary democracies, cabinet posts in coalition governments are allotted to the parties in proportion to their seats in parliament, with the "leading" party (if there is one) getting the more important posts. If Parties 1, 2, and 3 elect 26 per cent, 26 per cent, and 48 per cent of parliament respectively, this yields Roth's game; the parameter p is[21] at most 35, and may be much smaller (depending on the importance of the "important" posts). Roth suggests that the smaller parties will *necessarily* form the government, that the large party is not only weaker than the small parties, but is actually completely powerless. But many people would say the opposite, that the large party has *more* power than the smaller parties.

For a market story, we turn to Shafer's example [**46**]. There are two goods: Agents 1, 2, 3 have endowments $(1-\varepsilon, 0)$, $(0, 1-\varepsilon)$, $(\varepsilon, \varepsilon)$, and utility functions $\sqrt{xy}$, $\sqrt{xy}$, $(x+y)/2$, respectively, where[22] $0<\varepsilon<1$. The NTU value gives 3 at least $1/6$, even when ε is small.

The example is basically similar to Roth's, though less clear-cut.[23] What lends it credibility is the market context, in which endowments are more "visible" than utilities. In the previous story, 3 appeared as a large powerful political party; here he appears as a miserable peddler, with hardly any goods.

Shafer may well have a point. Game theory, as well as economics, typically provides multiple solutions. But game theoretic concepts apply to a "purified" or "processed" version of the original situation, such as the coalitional[24] or strategic[25] form. The processing removes some of the "glue" that gives the situation coherence; to choose among the multiple solutions, it may be necessary to restore some of this glue, to go back and look at the "raw," original situation.

[19] This may not be the case in more concrete situations. See the next section.

[20] If $p=0$, then it appears that in most natural bargaining models, all perfect EP's do lead to $(1/2, 1/2, 0)$ (though again, ordinary EP's need not). But in that case, $(1/2, 1/2, 0)$ is also an NTU value (set $\lambda_1=\lambda_2=1, \lambda_3=0$).

[21] We are abstracting away from political ideology, i.e., assuming that all that matters is influence in the government.

[22] Shafer also considers $\varepsilon=0$, but we do not; see note 20.

[23] In Roth's example, 1 and 2 can achieve an outcome that is preferred by each of them to any other feasible outcome. Therefore if they meet before either one meets 3, it is a foregone conclusion that they will form a coalition, because there is no room for any argument between them. This is not true in Shafer's example; if they meet, 1 and 2 may argue, perhaps even disagree. This weakens them in itself, and also because the uncertain outcome if they meet diminishes whatever resolve they may have had to refuse offers from 3.

[24] "Characteristic function."

[25] "Normal."

T.C. Schelling [**44**] used the term "focal point" for an EP suggested by the particular context of a game. Suppose that two people arrange to meet, but neglect to specify where. If in the past they have frequently met at a certain bar, it will be natural for them to seek each other there; though there is nothing in the mathematical structure of the game that distinguishes this EP from any other, their mutual expectations reinforce each other and make it a likely outcome. When nations negotiate boundaries, rivers and watersheds are focal points. In deciding the level of poison gas or nuclear weapons that international convention tolerates in war, the zero level suggests itself as a focal point, even though one or both sides might prefer a different level.

In the above political context, history, custom, public opinion, and the sheer size of 3 generate a perception that it will probably lead the government; once there, this perception reinforces itself[26] and becomes a focal point. In Shafer's context, 3's puniness generates the opposite perception, that he will be excluded from the trading; it, too, reinforces itself and becomes a focal point.

The NTU value is "context-neutral." Based on the coalitional worth function only, it cannot take into account the peculiar features of each realization of this function. In discussing the situation, one can stay away from special contexts, as Roth did, and as we did in Sections 3-7. But if one does wish to tell stories, then more than one can be told; and again it turns out that on the whole, the outcome $(1/3, 1/3, 1/3)$ is not a bad reflection of the qualitative situation.

9. THE CONCEPTUAL BACKGROUND OF ROTH'S SOLUTION

Underlying Roth's solution is the idea of domination, the same idea as that which underlies the core and the NM (von Neumann-Morgenstern) solution [**33**]. Recall that an outcome x dominates an outcome y iff there is a coalition that can achieve x, each of whose members prefers x to y. In Roth's game, $(1/2, 1/2, 0)$ dominates all other outcomes; it is the only point in the core, and constitutes the only NM solution.

But domination, though unquestionably of fundamental importance, does not have the elemental persuasiveness of simple rationality (i.e., individual utility maximization). It is based on the principle of cooperation—that people should always act jointly to further common interests; and this goes substantially beyond rationality, which says only that an individual should always act in his own interests.[27] *Domination involves relying on others to cooperate*, and as we have seen above, that is not always the way to maximize utility.

Value theory, on the other hand, is not based on domination. The value of an individual is a kind of index or average, based on the strength of the coalitions of which he is a member, and of those of which he is not a member. No attempt is made to predict which coalitions will form; all coalitions are considered. This fits in well with the Nash program, which in Roth's game also leads to all coalitions.

In the context of cooperative games, the principle of cooperation may sound quite reasonable, and solution concepts based on domination certainly merit study. But the term "cooperative" is usually taken to mean only that players *can* make binding agreements, without any implication about what they *should* or are expected to do. The NTU value fits in well with this broader view, and certainly it, too, merits study.

[26] As described in Sections 3, 5, and 6.

[27] It might be argued that rationality implies the principle of cooperation, since cooperation increases the utility of each individual in the group. But this argument is circular. The principle of cooperation will not be effective unless it is adopted by all involved; in general, it will not be rational for a single individual to adopt it. Moreover, it is by no means clear that it would be good for Society as a whole to adopt it. Thieves and murderers can also cooperate; in Roth's game, cooperation between 1 and 2 would be great for them, but not so good for 3.

We conclude by doffing our hat to Roth and Shafer. Their examples are ingenious, thought provoking, and far from transparent; and there is no doubt that they have led to a deeper understanding of the NTU value and of cooperative games in general.

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Manuscript received February, 1983; final revision received July, 1984.

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