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There are two stages and three players in the game. In the first stage, two players join into a coalition, with one representing the other, while the third player is left by himself. In the second stage, all three players join into one coalition with either the third player heading the group, or the representative from the previous stage leading the entire pack.

Some notation: the players are $P1$, $P2$, and $P3$. When $P1$ accepts $P2$ to be his agent, we denote the coalition by $P21$, and from this point on, $P2$ will represent $P1$. Write Pi for a generic player or coalition, where i may be one or two numbers. To denote the expected payoff for Pi, we write $UiBjRk$ (Utility of i given By j Representing k), where the coalition Pj represents Pk. Some examples of this:

1. The expected utility for $P3$ if he allows $P2$ to represent him is $U3B2R3$.

2. When $P3$ accepts the coalition $P21$ to represent him, his utility is $U3B21R3$.

   This should be distinguished from the case

3. when the coalition $P13$ accepts $P2$ as their agent. In this case, the utility of Pi is $UiB2R13$.

The probability that a player or coalition Pi accepts Pk is denoted by $AiFk$ (probability of Acceptance of i For k). So in the first round, the probability that $P1$ chooses $P2$ to represent him is $A1F2$. For the first round, instead of $AiFi$, we write Ni (No acceptance) with the obvious rule that $AiFj + AiFk + Ni = 1$ $(j \neq k \neq i)$.

Each coalition has a demand, denoted by Di. So coalition $P12$ demands $D12$ if player $P3$ is to be the representative.

In the first stage, if no player chooses to have another represent him, the game is allowed to repeat with probability $(1 - \varepsilon_4)$. If the game is not allowed to repeat, it ends with a zero payoff for each player. If more than one player asks to be represented, a uniform random event decides which player is represented. So at the end of stage one, either the players all have zero payoff, or one player is alone and another is leading his two-man coalition.

A similar procedure occurs in the second stage. Should neither the coalition nor the lone player want to accept the other as the representative, the game ends with probability ε_5. In this case, the lone player, Pi, receives a zero payoff, and the other two receive Bi each. (Other notation: other two receive $\frac{Bi}{2}$ each.) So if coalition $P12$ does not accept $P3$ and vice versa, then the payoff vector is $(B3, B3, 0)$.

The probabilities are related to the demands and utilities in the following way.

For coalitions Pi and Pj,

$$AiFj = \frac{\exp\left(\frac{UiBjRi-Di}{\varepsilon_i}\right)}{1+\exp\left(\frac{UiBjRi-Di}{\varepsilon_i}\right)}, \tag{1}$$

where ε_i is a constant, determining the sensitivity of the demand. If ε_i is close to zero, the slightest difference between the utility given and amount demanded drives the probability to zero or one, if the difference is negative or positive, respectively. The bigger ε_i, the less this difference is felt in the overall probability.

We now work backwards to determine expected utilities in the second stage.

Say we have come to stage two, and in the first stage, player $P1$ accepted $P2$ to be his representative.