MATH 201 Spring '01 Final Exam

1. *(20 points)*

   Consider the points

   $$A = (2, 3, 2) \qquad B = (4, 1, 0) \qquad C = (-1, 2, 0) \qquad D = (5, 4, -2)$$

   (a) Find an equation for the plane P that contains points A and B and is parallel to the line L through C and D.

   (b) Find the distance from the line L to the plane P.

2. *(16 points)*

   Find and classify the critical points (local max, local min, saddle points) of $f(x, y) = x^3y - 3xy + y^2$.

3. *(16 points)*

   Find the point of the surface $z = xy + 1$ that is closest to the origin.

4. *(16 points)*

   (a) Find an equation for the plane through $(1, 1, 1)$ that is normal to the twisted cubic $(x, y, z) = (t, t^2, t^3)$ at that point.

   (b) Find an equation for the plane tangent to the paraboloid $z = 2x^2 + 3y^2$ and simultaneously parallel to the plane $4x - 3y - z = 10$.

5. *(20 points)*

   Compute the volume of the part of the paraboloid $z = x^2+y^2$ contained inside the sphere $x^2+y^2+z^2 = 2$.

6. *(20 points)*

Let R be the region in the first quadrant of the xy-plane bounded by the hyperbolas $xy = 1$ and $xy = 9$ as well as by the lines $y = x$ and $y = 4x$.

(a) Find a mapping T from the $uv-$plane to the $xy-$plane such that R is the image of the rectangle with vertices $(1,1), (9,1), (9,4)$ and $(1,4)$.

(b) Compute

$$\int_R \left(\sqrt{\frac{y}{x}} + \sqrt{xy}\right) dA$$

7. *(28 points)*

Compute the circulation $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the following vector fields and curves C.

(a) $\mathbf{F} = \langle 2xy + x, xy - y \rangle$, and the unit square C with vertices $(0,0), (0,1), (1,1)$ and $(1,0)$, oriented counterclockwise.

(b) $\mathbf{F} = f\frac{\partial f}{\partial x}\mathbf{i} + f\frac{\partial f}{\partial y}\mathbf{j}$ where f is a smooth function defined everywhere on the $xy-$plane, and C is a closed curve in the $xy-$plane.

8. *(20 points)*

Compute the flux $\int_C \mathbf{F} \cdot \mathbf{n} ds$ for the vector field $\mathbf{F} = (5x\mathbf{i} + 5y\mathbf{j})/(x^2 + y^2)$ and the ellipse C with equation $x^2/9 + y^2/4 = 1$. Justify your answer.

9. *(24 points)*

Verify Stokes' theorem if $\mathbf{F} = \langle y, -x, 0 \rangle$ and C is the circle in the xy-plane that bounds the part of surface $x^2 + y^2 + z = 9$ with $z \geq 0$ (notice that the surface is not a hemisphere).

10. *(20 points)*

Compute the flux integral $\int_S \mathbf{F} \cdot \mathbf{n} dS$ of the vector field $\mathbf{F}(x,y,z) = (x^3, y^3, z^3)$ across the sphere $x^2 + y^2 + z^2 = 9$.