

Princeton-Rider Workshop **Abstracts**

(May 30, 2017)

Speaker: Anton Ayzenberg, Steklov Mathematical Institute and Higher School of Economics

Date: Tuesday, 5/30/17

Time and location: 1:35PM, RU Room 102

Title: TORIC ORIGAMI STRUCTURES ON QUASITORIC MANIFOLDS AND FATNESS OF 3-POLYTOPES.

Abstract: Toric origami manifold is a generalization of a symplectic toric manifold: it is a manifold with torus action and a compatible 2-form degenerating in a nice, controllable way. In the same way as symplectic toric manifolds are classified by Delzant polytopes, symplectic toric manifolds are classified by certain collections of Delzant polytopes, called origami templates. We are interested in the following question: are there quasitoric manifolds which are not toric origami?

It happens that in dimension 6 there are plenty of them: we introduce a certain parameter of a polytope: the fatness, and show that fat polytopes admit a quasitoric manifold which is not toric origami. To show that there exist fat polytopes one can use an isoperimetric inequality on a sphere. Moreover, some recent developments in the theory of large random planar maps show that most simple 3-polytopes are fat. This means, that most simple 3-polytopes admit a quasitoric manifold which is not toric origami.

This is a joint work with M.Masuda, S.Park and H.Zeng.

REFERENCES

- [1] A. Ayzenberg, M. Masuda, S. Park, H. Zeng, *Toric origami structures on quasitoric manifolds*, Proc. Steklov Inst. Math. 288:1 (2015), 10-28.
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Speaker: Ciprian Borcea, Rider University

Date: Tuesday, 5/30/17

Time and location: 5:10PM, PU Jadwin A10

Title: AUXETIC SPACETIMES

Abstract: The notion of auxetic behavior comes from materials science and refers to lateral widening upon stretching. A strictly geometrical approach to auxetic deformations of periodic frameworks, introduced by C. Borcea and I. Streinu, revealed analogies with cone structures in spacetimes of arbitrary dimension. In this lecture, we define auxetic spacetimes and explain the role of spectrahedral and hyperbolicity cones.

Speaker: Frédéric Bosio, Université de Poitiers

Date: Tuesday, 5/30/17

Time and location: 2:05PM, PU Jadwin A10

Title: ON DIFFEOMORPHIC MOMENT-ANGLE MANIFOLDS.

Abstract: In this talk, we investigate the moment-angle manifolds, a particular case of polyhedral product, that is associated to any simple polytope. We focus on the problem of determining when two polytopes give rise to the same manifolds.

We introduce and put emphasis on a notion we call *puzzle-equivalence*, and give a couple of applications of this notion.

Speaker: Li Cai, Xi'an Jiaotong-Liverpool University

Date: Wednesday, 5/31/17

Time and location: 2:50PM, RU Room 102

Title: ON THE HOMOLOGY OF A SMALL COVER.

Abstract: In this talk I will present our results on the integral homology of a small cover. In particular, we give a necessary and sufficient condition that, all torsion elements in homology have order two. Several examples in low dimensions will also be presented.

Speaker: John Conway, Princeton University

Date: Friday, 6/2/17

Time and location: 2:50PM, PU Jadwin A10

Title: THE SYMMETRICALLY TWICE-BORDERED MÖBIUS STRIP.

Abstract: In this talk, I shall describe the strange object you see in my hand.

Speaker: Alastair Darby, Xi'an Jiaotong-Liverpool University

Date: Thursday, 6/1/17

Time and location: 2:50PM, RU Room 102

Title: CLUSTER ALGEBRAS AND ALGEBRAIC PL-INVARIANTS.

Abstract: In this joint work with Zhi Lü we attempt to use the theory of cluster algebras to find algebraic invariants for piecewise linear manifolds. This is inspired by the work of Fomin, Shapiro and Thurston where they encode equivalence classes of triangulated surfaces using cluster algebras. Using generalisations and adaptations of this theory we shed a modern light on a classical subject.

Speaker: Nikolai Erokhovets, Lomonosov Moscow State University

Date: Monday, 5/29/17

Time and location: 10:00AM, PU Jadwin A10

Title: TORIC TOPOLOGY AND COMBINATORICS OF FULLERENES AND POGORELOV POLYTOPES.

Abstract: A (*mathematical*) fullerene is a simple convex 3-polytope with all facets 5- and 6-gons. A k -belt of a simple 3-polytope is a cyclic sequence of facets with empty common intersection such that facets intersect if and only if they follow each other. It is known that fullerenes has no 3- and 4-belts. Results by A.V. Pogorelov and E.M. Andreev imply that this condition is the criterion for a simple 3-polytope to be realized in Lobachevsky (hyperbolic) space as a bounded polytope with right angles. Such polytopes we call Pogorelov polytopes. It follows from results by F. Fan, J. Ma and X. Wang [1] that two Pogorelov polytopes P and Q are combinatorially equivalent, if the graded cohomology rings $H^*(\mathcal{Z}_P)$ and $H^*(\mathcal{Z}_Q)$ of moment-angle manifolds are isomorphic. We will discuss recent result by V.M. Buchstaber, N.Yu. Erokhovets, M. Masuda, T.E. Panov and S. Park [2] that for Pogorelov polytopes characteristic pairs (P_1, Λ_1) and (P_2, Λ_2) are equivalent, if the graded cohomology rings $H^*(M(P_1, \Lambda_1))$ and $H^*(M(P_2, \Lambda_2))$ of quasitoric manifolds are isomorphic. The example of a Pogorelov polytope is given by a k -barrel – a polytope with surface glued from two patches, each consisting of a k -gon surrounded by a k -belt of 5-gons. Results by T. Inoue [3] imply that any Pogorelov polytope can be combinatorially obtained from k -barrels by a sequence of (s, k) -truncations (cutting off s consequent edges of a k -gon by a single plane), $2 \leq s \leq k - 4$, and connected sums along k -gonal faces (combinatorial analog of glueing two polytopes along k -gons perpendicular to adjacent facets). We prove that any Pogorelov polytope except for k -barrels can be obtained from the 5- or 6-barrel by $(2, k)$ -truncations, $k \geq 6$, and connected sums with 5-barrels along pentagons. In the case of fullerenes we prove a stronger result. Let $(2, k; m_1, m_2)$ -truncation be a $(2, k)$ -truncation that cuts off two edges intersecting an m_1 -gon and an m_2 -gon by vertices different from the common vertex. There is an infinite family of connected sums of 5-barrels along pentagons surrounded by pentagons called $(5, 0)$ -nanotubes. We prove that any fullerene except for the 5-barrel and the $(5, 0)$ -nanotubes can be obtained from the 6-barrel by a sequence of $(2, 6; 5, 5)$ -, $(2, 6; 5, 6)$ -, $(2, 7; 5, 6)$ -, $(2, 7; 5, 5)$ -truncations such that all intermediate polytopes are either fullerenes or Pogorelov polytopes with facets 5-, 6- and one 7-gon with the 7-gon adjacent to some 5-gon.

These results are obtained in joint work with V.M. Buchstaber [4].

This research is partially supported by the Young Russian Mathematics award and the RFBR grants 17-01-00671 and 16-51-55017.

REFERENCES

- [1] F. Fan, J. Ma, X. Wang, *B-Rigidity of flag 2-spheres without 4-belt*, arXiv:1511.03624.
 - [2] V.M. Buchstaber, N.Yu. Erokhovets, M. Masuda, T.E. Panov, S. Park, *cohomological rigidity of manifolds defined by right-angled 3-dimensional polytopes*, Russian Math. Surveys, **72**:2 (2017), arXiv:1610.07575v3.
 - [3] T. Inoue, *Organizing volumes of right-angled hyperbolic polyhedra*, Algebraic & Geometric Topology, **8** (2008), 1523–1565.
 - [4] V.M. Buchstaber, N.Yu. Erokhovets, *Construction of families of three-dimensional polytopes, characteristic patches of fullerenes and Pogorelov polytopes*, Izvestiya: Mathematics, **81**:5 (2017).
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Speaker: Feifei Fan, Chinese Academy of Sciences

Date: Monday, 5/29/17

Time and location: 5:55PM, PU Jadwin A10

Title: COHOMOLOGICAL RIGIDITY OF A CLASS OF MOMENT-ANGLE MANIFOLDS AND TORIC (QUASITORIC) MANIFOLDS.

Abstract: Buchstaber, Erochovets, Masuda, Panov and Park prove a result that, a class of 6-dimensional toric (quasitoric) manifolds M has cohomological rigidity. More specifically, if the corresponding 3-dimensional simplicial fans (resp. simple polytopes) satisfy the no \triangle and \square condition, then the equivariant diffeomorphism class of M is determined by its (ordinary) cohomology ring. We also prove the same result in another way by using our result about the cohomological rigidity of moment-angle manifolds and Taylor resolutions. Moreover, we get a generalization of this result for higher dimensional toric (quasitoric) manifolds along this line of reasoning. More specifically, for any simplicial n -sphere ($n \geq 3$) K (resp. simple polytope P) which is flag and satisfies the separating circuit condition (a combinatorial analog of the no \square condition in the 3-dim case), the moment-angle manifold Z_K (resp. the toric manifold M_K and quasitoric manifold M_P defined on it) is cohomological rigid.

This is joint work with Jun Ma and Xiangjun Wang.

Speaker: Jelena Grbić, Southampton University

Date: Wednesday, 5/31/17

Time and location: 10:00AM, RU Room 102

Title: CONFIGURATION SPACES AND POLYHEDRAL PRODUCTS.

Abstract: In the talk I shall aim to present the most general combinatorial conditions under which a moment-angle complex $(D^2, S^1)^K$ is a co- H -space, thus splitting unstably in terms of its full subcomplexes. In this way Piotr Beben and I studied to which extent the conjecture holds that a moment-angle complex over a Golod simplicial complex is a co- H -space. Our main tool is a certain generalisation of the theory of labelled configuration spaces.

Speaker: Vladimir Grujić, Belgrade University

Date: Monday, 5/29/17

Time and location: 5:10PM, PU Jadwin A10

Title: COMBINATORICS OF GENERALIZED PERMUTOHEDRA.

Abstract: Generalized permutohedra are convex polytopes resulting by deformations of the standard permutohedron. Some important classes of convex polytopes are generalized permutohedra: graphic zonotopes, nestohedra and matroid base polytopes. A generalized permutohedron Q determines a certain quasisymmetric enumerator function $F_q(Q)$ that contains complete information of the f -vector of the polytope Q . This enumerator function is a common refinement of Stanley's chromatic symmetric function of a graph and the Billera–Jia–Reiner quasisymmetric function of a matroid. The combinatorics of generalized permutohedra determines a combinatorial Hopf algebra structure on graphs, building sets and matroids which produces this enumerator function. This talk is based on preprints [1], [2].

REFERENCES

- [1] V. Grujić, *Quasisymmetric functions for nestohedra*, to appear in SIAM J. Discrete Math. arXiv:1409.1420
 - [2] V. Grujić, M. Pešović, *Face enumeration on matroid base polytopes*, arXiv:1704.06715
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Speaker: Sho Hasui, Osaka Prefecture University

Date: Wednesday, 5/31/17

Time and location: 5:55PM, RU Room 102

Title: ON THE STABLE COHOMOLOGICAL RIGIDITY OF QUASITORIC MANIFOLDS.

Abstract: In this talk we consider the stable homotopy version of the cohomological rigidity problem for quasitoric manifolds, which asks whether two quasitoric manifolds with the isomorphic cohomology rings are stably homotopy equivalent or not. We give a criterion for the stable cohomological rigidity to hold after p -localization.

This is joint work with Daisuke Kishimoto.

Speaker: Hiroaki Ishida, Kagoshima University

Date: Tuesday, 5/30/17

Time and location: 2:50PM, PU Jadwin A10

Title: DE RHAM AND DOLBEAULT MODELS OF MOMENT-ANGLE MANIFOLDS.

Abstract: A differential graded algebra A^* is called a (de Rham) model of a smooth manifold M if A^* is quasi-isomorphic to the de Rham complex of M . A differential bigraded algebra $A^{*,*}$ is called a Dolbeault model of a complex manifold M if $A^{*,*}$ is quasi-isomorphic to the Dolbeault complex of M . In this talk we give de Rham and Dolbeault models of certain moment-angle manifolds.

Speaker: Shizuo Kaji, Yamaguchi University

Date: Monday, 5/29/17

Time and location: 2:50PM, PU Jadwin A10

Title: REPRESENTATIONS ON REAL TORIC MANIFOLDS.

Abstract: When a group G acts on a manifold M , the (co)homology of M is equipped with a G -module structure. One of the central questions in representation theory asks to realise a given G -module geometrically in this manner. In this talk, we consider finite group actions on real toric manifolds combinatorially through the correspondence between real toric manifolds and simplicial complexes with characteristic matrices. In particular, we see interesting representations of (signed) permutations appear on the homology of certain real toric manifolds.

This is joint work with Soojin Cho and Suyoung Choi.

Speaker: Lukas Katthän, University of Minnesota

Date: Monday, 5/29/17

Time and location: 5:55PM, PU Jadwin A10

Title: THE GOLOD PROPERTY OF STANLEY-REISNER RINGS.

Abstract: A graded (or local) ring R is called Golod if all products and Massey products on its Tor-algebra vanish. If R is a Stanley-Reisner ring, then its Tor-algebra is isomorphic to the cohomology ring of the corresponding Moment-Angle complex, and so the Tor-algebra gives a way to study the product and Massey products on the latter. In this talk, I will discuss several criteria for deciding whether a simplicial complex is Golod. Moreover, I will present an example where the Golod property depends on the underlying field, and an example where the Koszul homology has only trivial products but a nontrivial ternary Massey product.

Speaker: [Daisuki Kishimoto](#), Kyoto University

Date: Monday, 5/29/17

Time and location: 11:30AM, PU Jadwin A10

Title: RIGHT-ANGLED COXETER QUANDLES AND POLYHEDRAL PRODUCTS.

Abstract: The set of reflections of a Coxeter group W yields a new group $\text{Ad}(Q_W)$ through a quandle Q_W , and it is shown by Akita that $\text{Ad}(Q_W)$ is an intermediate object between W and the associated Artin group A_W . So one naturally asks which properties of W are inherited by $\text{Ad}(Q_W)$. It is known that W is right-angled if and only if BW is given by a certain polyhedral product. We show that the right-angularity of W is inherited by $\text{Ad}(Q_W)$ in such a way that its classifying space is also given by a polyhedral product. We then discuss its applications.

Speaker: [Shintaro Kuroki](#), Okayama University of Science

Date: Wednesday, 5/31/17

Time and location: 5:10PM, RU Room 102

Title: EXTENDED ACTIONS OF QUASITORIC MANIFOLDS AND J-CONSTRUCTIONS OF POLYTOPES.

Abstract: Bahri, Bendersky, Cohen and Gitler introduce a construction of infinite families of quasitoric manifolds from the given quasitoric manifold, called a J-construction. This construction is a purely combinatorial construction. So it may be natural to ask what the geometric meaning of J-construction is. In this talk, we introduce that the J-construction (combinatorics) naturally appears in the context of extended actions (geometry).

Speaker: [Hideya Kuwata](#), Osaka City University

Date: Friday, 6/2/17

Time and location: 2:05PM, PU Jadwin A10

Title: CLASSIFICATION OF TORIC MANIFOLDS OVER AN n -CUBE WITH ONE VERTEX CUT.

Abstract: Toric manifolds (= compact smooth toric varieties) over an n -cube are known as Bott manifolds (or Bott towers) and their topology is well studied. The blow up of Bott manifolds at a fixed point provides toric manifolds over an n -cube with one vertex and they are all projective since so are Bott manifolds. On the other hand, Oda's 3-fold, which is known as the simplest non-projective toric manifold, is over a 3-cube with one vertex cut. In this talk, we classify toric manifolds over an n -cube with one vertex cut as varieties and also as smooth manifolds. It turns out that there are many non-projective toric manifolds over an n -cube with one vertex cut (we can even count them in each dimension) but surprisingly they are all diffeomorphic.

This is joint work with Sho Hasui, Mikiya Masuda and Seonjeong Park.

Speaker: Eunjeong Lee, Korean Advanced Institute of Science and Technology

Date: Friday, 6/2/17

Time and location: 3:35PM, PU Jadwin A10

Title: BOTT TOWERS, GENERALIZED BOTT TOWERS, AND FLAGGED BOTT TOWERS.

Abstract: For a given smooth manifold, one could make a new manifold by taking a fiber bundle on it. In this talk, we study three of those fibrations—Bott towers, generalized Bott towers, and flagged Bott towers. By considering an iterated sequence of $\mathbb{C}P^1$ -fiber bundles such that each stage of which comes from a sum of two holomorphic line bundles, we get a Bott tower. A Bott tower is a smooth projective toric variety. Instead of considering $\mathbb{C}P^1$ -fibrations, we can consider an iterated sequence of $\mathbb{C}P^{n_j}$ -fiber bundles, which are projectivization of Whitney sum of holomorphic line bundles. This defines a generalized Bott tower, which is a projective toric variety. In this talk, we consider an another extended notion, called flagged Bott tower, which is a sequence of $Flag(n_j + 1)$ -fiber bundles. A flagged Bott tower is not a toric variety but it has a torus action. In this talk, we study the relation between generalized Bott towers and flagged Bott towers.

This talk is based on an on-going project with Shintarô Kuroki, Jongbaek Song, and Dong Youp Suh.

Speaker: Ivan Limonchenko, Fudan University

Date: Friday, 6/2/17

Time and location: 11:30PM, PU Jadwin A10

Title: ON THE RATIONAL FORMALITY OF POLYHEDRAL PRODUCTS OVER FLAG SIMPLE POLYTOPES.

Abstract: It is a classical result of rational homotopy theory that a nontrivial higher order Massey product in cohomology is an obstruction to formality of a space. T. Panov and N. Ray proved that quasitoric manifolds are formal spaces; projective toric manifolds are Kähler and thus formal due to P. Deligne, P. Griffiths, J. Morgan and D. Sullivan. On the other hand, I. Baskakov found a class of triangulated spheres K for which \mathcal{Z}_K are non-formal manifolds having a nontrivial triple Massey product of 3-dimensional classes in cohomology. Later on, such case was totally described in combinatorial terms for any simplicial complex K by G. Denham and A. Suciu.

In this talk we are going to discuss higher Massey products in cohomology and formality of generalized moment-angle manifolds arising from a wide class of flag simple polytopes among which are flag nestohedra (graph-associahedra) and Pogorelov class polytopes (fullerenes).

Speaker: Zhi Lü, Fudan University

Date: Friday, 6/2/17

Time and location: 5:10PM, PU Jadwin A10

Title: LICKORISH TYPE CLASSIFICATION OF CLOSED MANIFOLDS OVER SIMPLE POLYTOPES.

Abstract: Each simple polytope P with m facets gives rise to a family of smooth closed manifolds, each of which can be obtained from the (real) moment-angle manifold $\mathcal{Z}_P(\mathbb{R}\mathcal{Z}_P)$ over P via the free actions of the subgroups of T^m on \mathcal{Z}_P (\mathbb{Z}_2^m on $\mathbb{R}\mathcal{Z}_P$). We will be concerned with the Lickorish type classification of all smooth closed manifolds given the free actions on associated real moment-angle manifolds over all simple polytopes. This talk will show some progress.

Speaker: Jun Ma, Capital Normal University

Date: Monday, 5/29/17

Time and location: 2:05PM, PU Jadwin A10

Title: THE FIXED POINT DATA AND EQUIVARIANT CHERN NUMBERS.

Abstract: First, we follow the approach of tom Dieck to consider the unitary T^k -equivariant cobordism and reprove equivariant Chern numbers determine the bordism class of a unitary T^k -manifold. Then for given family of connected closed unitary manifolds with trivial T^k action and T^k complex vector bundles of them, we find necessary and sufficient conditions for the existence of a unitary T^k manifold with the given fixed point data by using equivariant Chern numbers.

As an application, we deal with the generalized Kosniowski conjecture. We prove that for a $2n$ -dimensional closed unitary manifold with T^{n-1} action which has only isolated fixed points, if M does not bound equivariantly, the number of fixed points is greater than a linear function of n .

This is a joint work with Lü Zhi and Wang Wei for the first part, and Wen Shiyun for the generalized Kosniowski conjecture.

Speaker: Mikiya Masuda, Osaka City University

Date: Thursday, 6/1/17

Time and location: 10:00AM, RU Room 102

Title: VOLUME POLYNOMIALS OF REGULAR NILPOTENT HESSENBERG VARIETIES.

Abstract: For a homogeneous polynomial P in n variables x_1, \dots, x_n ,

$$\text{Ann}(P) := \{f \in \mathbb{R}[x_1, \dots, x_n] \mid f(\partial/\partial x_1, \dots, \partial/\partial x_n)(P) = 0\}$$

is a graded ideal of the polynomial ring $\mathbb{R}[x_1, \dots, x_n]$ and the quotient

$$A(P) := \mathbb{R}[x_1, \dots, x_n]/\text{Ann}(P)$$

is a Poincaré duality algebra. Conversely, any Poincaré duality algebra A generated by degree one elements can be obtained this way, i.e. there exists a homogeneous polynomial P such that $A \cong A(P)$. The polynomial P is sometimes called the *volume polynomial* of the algebra A because if the algebra A is the cohomology ring of a projective toric manifold, then the polynomial P is related to the volumes of polytopes associated to the projective toric manifold.

In this talk we discuss about the volume polynomials (of the cohomology rings) of regular nilpotent Hessenberg varieties, a family of subvarieties of the flag variety G/B . This family contains the flag variety itself and Peterson variety which is in some sense minimal among regular nilpotent Hessenberg varieties and closely related to the toric manifold called permutohedral variety.

This is a part of joint work with Takuro Abe, Tatsuya Horiguchi, Satoshi Murai and Takashi Sato: *Hessenberg varieties and hyperplane arrangements*, arXiv:1611.00269.

Speaker: [Santiago López de Medrano](#), Universidad Nacional Autónoma de México

Date: Friday, 6/2/17

Time and location: 10:45AM, PU Jadwin A10

Title: SOME MOMENT-ANGLE MANIFOLDS AND POLYTOPES WITH DIHEDRAL SYMMETRY.

Abstract: We present new results about certain intersections of quadrics with dihedral symmetry that generalize basic examples (LdM 1984, De la Vega-LdM 2012). The main problem is to know when they are transverse and, therefore, moment-angle manifolds.

The equations of these quadrics are built with rows of the $n \times n$ Vandermonde matrix with entries the n -th roots of unity (alias Discrete Fourier Transform matrix). In 2014 we made some conjectures about the minors of this matrix, related to the transversality question:

- (1) For n prime, all the minors of this matrix are different from zero.
- (2) For n composite, a minor is 0 if, and only if, the complementary minor is 0.

Since 2016 we know that they are both true: (1) has had several independent proofs: Chebotariov (1926), Dieudonné (1970) and Tao (2003), among others. We proved (2) ourselves together with partial extensions of (1) to the composite case. All this already gives an interesting duality and many examples.

This is joint work with Matthias Franz.

Speaker: [Jason McCullough](#), Rider University

Date: Wednesday, 5/31/17

Time and location: 2:05PM, RU Room 102

Title: A SURVEY OF RECENT ADVANCES IN FREE RESOLUTIONS.

Abstract: There have been several breakthroughs in the theory of minimal free resolutions over a polynomial ring in the last 12 months. In my talk I will describe the recent solution to the Weak Horrocks Conjecture (and hence also the Toral Rank Conjecture) in all characteristics except 2 by Mark Walker, the positive answer to Stillman's Question (which asks for a bound on the projective dimension of ideals in terms of degrees of generators only) by Tigran Ananyan and Mel Hochster, and counterexamples to the Eisenbud-Goto Conjecture constructed by Irena Peeva and myself.

Speaker: Taras Panov, Moscow State University

Date: Friday, 6/2/17 **Time and location:** 10:00AM, PU Jadwin A10

Title: MANIFOLDS DEFINED BY RIGHT-ANGLED 3-DIMENSIONAL POLYTOPES.

Abstract: A combinatorial 3-dimensional polytope P can be realised in Lobachevsky 3-space with right dihedral angles if and only if it is simple, flag and does not have 4-belts of facets. This criterion was proved in the works of Pogorelov and Andreev of the 1960s. We refer to combinatorial 3-polytopes admitting a right-angled realisation in Lobachevsky 3-space as Pogorelov polytopes. The Pogorelov class contains all fullerenes, i.e. simple 3-polytopes with only 5-gonal and 6-gonal facets. There are two families of smooth manifolds associated with Pogorelov polytopes. The first family consists of 3-dimensional small covers of Pogorelov polytopes P , also known as hyperbolic 3-manifolds of Löbell type. These are aspherical 3-manifolds whose fundamental groups are certain finite abelian extensions of hyperbolic right-angled reflection groups in the facets of P . The second family consists of 6-dimensional quasitoric manifolds over Pogorelov polytopes. These are simply connected 6-manifolds with a 3-dimensional torus action and orbit space P . Our main result is that both families are cohomologically rigid, i.e. two manifolds M and M' from either of the families are diffeomorphic if and only if their cohomology rings are isomorphic. We also prove that a cohomology ring isomorphism implies an equivalence of characteristic pairs; in particular, the corresponding polytopes P and P' are combinatorially equivalent. These results are intertwined with the classical subjects of geometry and topology, such as combinatorics of 3-polytopes, the Four Colour Theorem, aspherical manifolds, diffeomorphism classification of 6-manifolds and invariance of Pontryagin classes. The proofs use techniques of toric topology.

This is a joint work with V.Buchstaber, N.Erokhovets, M.Masuda and S.Park, see arXiv:1610.07575.

Speaker: Seonjeong Park, Osaka City University

Date: Monday, 5/29/17

Time and location: 10:45AM, PU Jadwin A10

Title: REAL TORIC MANIFOLDS AND SHELLABLE POSETS ARISING FROM GRAPHS.

Abstract: For a graph G , a poset $P_{G,C}^{even}$ of C -even subgraphs is firstly considered by S. Choi and H. Park to study the topology of a real toric manifold associated with a simple graph. In this talk, we extend their result to a graph allowing multiple edges. We will discuss the shellability of the posets of even-subgraphs and real toric manifolds associated with (nonsimple) graphs. At first, we completely characterize the graphs (allowing multiple edges) whose posets of the even subgraphs are always shellable. Secondly, we compute the rational Betti numbers of real toric manifolds related to path graphs, and then we get some interesting integer sequences from the rational Betti numbers.

This is joint work with Boram Park.

Speaker: Soumen Sarkar, Indian Institute of Technology Madras

Date: Monday, 5/29/17

Time and location: 3:35PM, PU Jadwin A10

Title: EQUIVARIANT AND INVARIANT TOPOLOGICAL COMPLEXITY.

Abstract: Topological complexity of the configuration space of a mechanical system was introduced by M. Farber in 2003 to estimate the complexity of a motion planning algorithm. There are several generalizations of this topological invariant. In this talk I will introduce two of these generalizations.

This is joint work with M. Bayeh.

Speaker: Takashi Sato, Osaka City University

Date: Friday, 6/2/17

Time and location: 5:55PM, PU Jadwin A10

Title: HESSENBERG VARIETIES AND HYPERPLANE ARRANGEMENTS.

Abstract: A Hessenberg variety is a subvariety of a flag variety determined by a “good” subset of the positive root system and an element of the corresponding Lie algebra. A subset of the positive root system gives a hyperplane arrangement in the Lie algebra of a maximal torus. Similarly to a flag variety, the chambers of this arrangement denote a cell decomposition of the (regular nilpotent) Hessenberg variety. By this relation between a Hessenberg variety and a hyperplane arrangement, we describe the cohomology ring of the (regular nilpotent) Hessenberg variety in terms of the subarrangement.

This is joint work with Takuro Abe, Tatsuya Horiguchi, Mikiya Masuda, and Satoshi Murai.

Speaker: Jongbaek Song, Korean Advanced Institute of Science and Technology

Date: Wednesday, 5/31/17

Time and location: 3:35PM, RU Room 102

Title: THE EQUIVARIANT COHOMOLOGY RING OF TORIC ORBIFOLDS.

Abstract: It is well known that the equivariant cohomology ring of a smooth projective toric variety is completely determined by the orbit space. A toric orbifold is a projective toric variety having orbifold singularities. Since it is rationally smooth, the equivariant cohomology with rational coefficients does not distinguish between a smooth projective toric variety and a toric orbifold which have the same orbit space. In this talk, we discuss the integral equivariant cohomology ring of a toric orbifold, and introduce how this idea can be extended to torus orbifolds.

This talk is based on the joint work with Anthony Bahri and Soumen Sarkar, and the ongoing project with Shintaro Kuroki and Alastair Darby.

Speaker: Mentor Stafa, Indiana University Purdue University Indianapolis

Date: Wednesday, 5/31/17

Time and location: 10:45AM, RU Room 102

Title: POLYHEDRAL PRODUCTS AND MONODROMY REPRESENTATIONS.

Abstract: We will use polyhedral products to obtain faithful representations of graph products of finite groups and direct products of finite groups into automorphisms of free groups $\text{Aut}(F_n)$ and outer automorphisms of free groups $\text{Out}(F_n)$, respectively, as well as faithful representations of products of finite groups into the linear groups $\text{SL}(n, \mathbb{Z})$ and $\text{GL}(n, \mathbb{Z})$. These representations are realized as monodromy representations.

Speaker: Donald Stanley, University of Regina

Date: Tuesday, 5/30/17

Time and location: 10:00AM, PU Jadwin A10

Title: CLATS AND INTEGRAL SULLIVAN MODELS.

Abstract: A CLAT is a CDGA over the rationals together with an integral lattice of maximal rank inside its cohomology. We can get a CLAT in two ways: from a CDGA, A over the integers, and from a space X . If this gives rise to an isomorphic CLAT then we say that A is an integral Sullivan model for X . Any finite dimensional CLAT comes from an integral CDGA, but this is not true for all CLATs. We are also interested in which CLATs come from spaces. We conjecture that if a space has no torsion in its cohomology, then it has an integral Sullivan model whose cohomology has no torsion, and can prove this in some special cases.

This is joint with work Jonathan Xiyuan Wang.

Speaker: Dong Youp Suh, Korean Advanced Institute of Science and Technology

Date: Wednesday, 5/31/17

Time and location: 11:30AM, RU Room 102

Title: ON FLAGGED BOTT-SAMELSON VARIETIES AND FLAGGED BOTT MANIFOLDS.

Abstract: A Bott-Samelson variety is a nonsingular algebraic variety X_I constructed from a semi-simple complex Lie group G , and topologically it is the total space of an iterated $\mathbb{C}P^1$ -fibrations. Note that there is an induced action of a maximal torus T of G on X_I from the left multiplication of T on G .

On the other hand a Bott-manifold B_n is a nonsingular toric variety constructed from the projectivizations of iterated sums of two complex line bundles. Like a Bott-Samelson variety, a Bott manifold is topologically the total space of an iterated $\mathbb{C}P^1$ -fibrations.

Grossberg and Karshon discovered a nice relation between Bott-Samelson varieties and Bott manifolds in 1994. Indeed, for a given Bott-Samelson variety X_I , there is a one parameter family of complex structures X_I^t on X so that $X_I^0 = X_I$ and $X_I^\infty = B_n$ for some Bott manifold B_n .

In this talk, we generalize these spaces and relations to a flagged Bott-Samelson variety $X_{\mathfrak{J}}$ and a flagged Bott manifold F_m , so that each $X_{\mathfrak{J}}$ deforms diffeomorphically into F_m , and they are diffeomorphic to the total space of an iterated flag space fibrations. In general, a flagged Bott tower F_m is not a toric variety, but there is a much higher-rank torus action than the maximal torus of G , so that with this torus action F_m is a GKM-manifold.

Speaker: Svjetlana Terzić, University of Montenegro

Date: Tuesday, 5/30/17

Time and location: 3:35PM, PU Jadwin A10

Title: THE UNIVERSAL EXAMPLE OF T^k - ACTION ON M^{2n} AND RELATED PROBLEMS.

Abstract: It is the well known problem to describe the structure of the smooth action of the compact torus T^k on the closed manifold M^{2n} assuming that there exists the almost moment map $\mu : M^{2n} \rightarrow P^k \subset \mathbb{R}^k$, where $k \leq n$. In the case when $k = n$ this problem is the background of toric and quasitoric geometry and topology. In the case when $k < n$ there are many open questions.

In our papers [1], [2] we develop the theory of $(2n, k)$ -manifolds and in this talk will be presented our results in the universal example of T^k -action on $M^{2n} = \mathbb{C}P^{N-1}$. Here $N = \binom{k}{l}$ and the T^k -action on \mathbb{C}^N is given by the l -th symmetric power of the standard T^k -representation in \mathbb{C}^k . This representation induces the T^k -action on $\mathbb{C}P^{N-1}$ for which P^k is the hypersimplex $\Delta_{k,l}$.

In this case the T^k -action can be extended to the action of the algebraic torus $(\mathbb{C}^*)^k$ on $\mathbb{C}P^{N-1}$ which leads to the problem of description of the toric manifolds in $\mathbb{C}P^{N-1}$ obtained as the closure of the $(\mathbb{C}^*)^k$ -orbits. Such toric manifolds in $\mathbb{C}P^{N-1}$ can be explicitly described by the system of algebraic equations. We demonstrate such description in the case $k = 2l$.

As an application we obtain the description of the toric manifolds in the complex Grassmann manifolds $G_{2l,l}(\mathbb{C})$ generated by the orbits of the canonical action of $(\mathbb{C}^*)^{2l}$.

This is a joint work with Victor M. Buchstaber.

REFERENCES

- [1] V. M. Buchstaber, S. Terzić, *(2n, k)-manifolds and applications*, Report No. 27/2014, Mathematisches Forschung Institut (2531 May 2014), Oberwolfach, 2014, 11-14.
- [2] V. M. Buchstaber, S. Terzić, *Topology and geometry of the canonical action of T^4 on the complex Grassmannian $G_{4,2}$ and the complex projective space $\mathbb{C}P^5$* , Mosc. Math. J. 16 (2016), no. 2, 237-273.

Speaker: Stephen Theriault, University of Southampton

Date: Tuesday, 5/30/17

Time and location: 11:30AM, PU Jadwin A10

Title: MOORE'S CONJECTURE FOR POLYHEDRAL PRODUCTS.

Abstract: In the early 1980's John Moore posed a conjecture which, if true, would establish a deep connection between the rational and torsion homotopy groups of simply-connected finite dimensional spaces. In this talk we show that Moore's Conjecture holds for generalised moment-angle complexes, and go on to characterise those polyhedral products which have finitely many rational homotopy groups.

This is joint work with Y. Hao and Q. Sun.

Speaker: Elizabeth Vidaurre, University of Rochester

Date: Thursday, 6/1/17

Time and location: 10:45AM, RU Room 102

Title: A HOMOTOPICAL GENERALIZATION OF THE BESTVINA-BRADY CONSTRUCTION.

Abstract: Given a map of simplicial complexes and a fixed monoidal pair (X, A) , we can define a map of their respective polyhedral products. In the case of a map f from a simplicial complex L to an $(m-1)$ -simplex and the pair $(S_1, *)$, we will describe a cellular chain complex of the homotopy fiber $\widetilde{T}_{L,f}$ as a module over the fundamental group of the base, \mathbb{Z}^m , stemming from work of Leary and Saadetoğlu. When $m = 1$, we recover Bestvina-Brady groups, which have been studied extensively. We will discuss some consequences to the homology of the space $\widetilde{T}_{L,f}$.

Speaker: Xiangjun Wang, Nankai University

Date: Tuesday, 5/30/17

Time and location: 10:45AM, PU Jadwin A10

Title: A MOMENT-ANGLE MANIFOLD WHOSE COHOMOLOGY IS NOT TORSION FREE.

Abstract: In this talk I will introduce a method to construct moment-angle manifolds whose cohomologies are not torsion free. I also give method to describe the corresponding simplicial sphere.

This is Gefe Wang and his group's work on toric topology.

Speaker: Haozhi Zeng, Fudan University

Date: Thursday, 6/1/17

Time and location: 11:30AM, RU Room 102

Title: ON THE TORSION IN THE COHOMOLOGY GROUPS OF TORIC ORBIFOLDS.

Abstract: The cohomology groups of toric orbifolds with rational coefficients are well known. In this talk we will discuss the torsion part of the cohomology groups of toric orbifolds with integral coefficients. For some special toric orbifolds, we will discuss necessary and sufficient conditions for the cohomology groups to be torsion-free. So, for these toric orbifolds, we can calculate their cohomology rings by a formula due to Bahri, Sarkar and Song.

This talk is based on the joint work with H. Kuwata and M. Masuda.
