Towards conformal invariance of 2-dimensional lattice models

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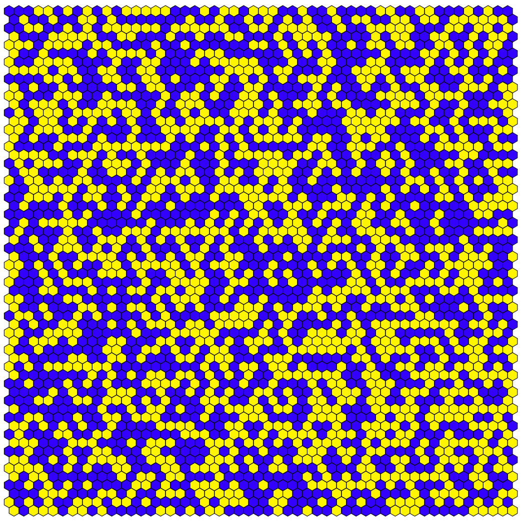
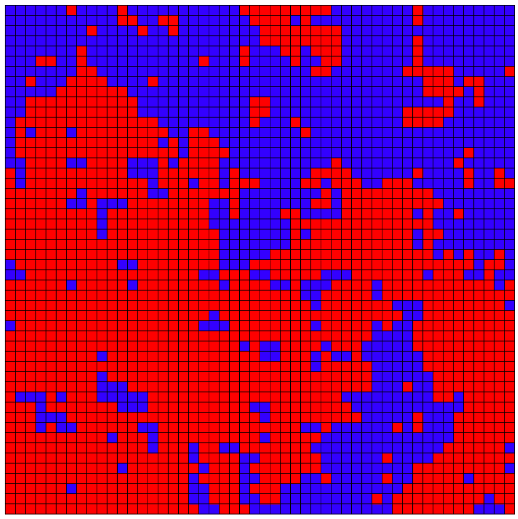
On the left there is a picture of the critical percolation: for every hexagon we toss a coin and color it blue or yellow depending on whether it came up heads or tails. Is there an up-down crossing of yellow hexagons? It is quite hard to see! The reason is that clusters (connected sets of the same color) are complicated fractals of dimension 91/48 (meaning that a cluster of diameter D on average has $\approx D^{91/48}$ hexagons).

On the right there is a picture of the Ising model at critical temperature: this time nearby squares tend to be of the same color. Again we observe complicated fractal sets (this time the dimension is 15/8).





Many similar results were predicted by physicists, and are now proved mathematically. The idea behind both physical and mathematical arguments is that the models involved are conformally invariant in the scaling limit (when we look at the picture from very far away).

We will discuss how discrete analytic functions arise in the lattice models and how this leads to the conformal invariance of the scaling limit.