I was invited to reflect on reason, certainty, and hope in science and especially in mathematics. This may seem as odd a topic to you as it did to me. It is a daunting challenge, but I am very grateful for the invitation. My experience of the radiant joy present at the Rimini Meeting of 2006 is unforgettable. Joy cannot be counterfeited; it is an unmistakable spiritual witness. It is good to be again in the warm ambience of Communion and Liberation.

Science must follow reason:

All that is science is rational.

But alas! many people maintain that therefore:

All that is rational is science.

This elementary violation of logic and reason, known as scientism, would be funny were it not so widely held by intelligent people and so pernicious. A scientist once said to me, "Ed, you are the most rational person I know. I find it incredible that you believe in God."

When I was a lad growing up in Rome I read two books that inoculated me against scientism. The first was *Freedom and the Spirit* by the Russian Orthodox philosopher Nicholas Berdyaev and the second was *Science and the Unseen World* by the English Quaker astrophysicist Arthur Stanley Eddington, writing in the 1920s. It meant a lot to me at that formative stage of my life to find a famous scientist speaking of his religious faith.

But a curious thing happened when I reread Eddington's book many years later: I noticed that most of the science was wrong! To take just one example, Eddington attributes the origin of the solar system to the near approach of another star, pulling out gas from our sun which then condensed into planets. Such an event would be extremely rare, implying that very, very few stars have planets. But today the discovery of new exoplanets is almost a weekly occurrence, and we even have pictures of some.

Anyone writing on science today must be prepared for the same fate: most of what is written will turn out to be wrong, and the more fundamental the science the more likely is it to fail the test of time. Science is rational, science makes progress, but there is no certainty in science. What is held to be true in science is (or should be) always held tentatively, subject to later emendation or even reversal. Scientific truth has a much shorter half-life than spiritual truth.

But in mathematics, many people assert, with its discipline of rigorous deduction, we have complete certainty of the truth of our discoveries: that the square on the hypotenuse is the sum of the squares on the other two sides is as true today as it was in ancient Greece. (See page 47 of Richard Fitzpatrick's superb bilingual online edition of Euclid's Elements.) But don't be too certain of this! The millennia-long history of mathematics can be read as a continual retreat from certainty.

Mathematics as we know it, with proofs, began with the Greeks and reached maturity with Euclid's *Elements*. Two remarks need to be made. First, Euclid set the standard for rigor but did not meet his own standard: the proof of the very first proposition in incorrect, relying on an intuitive assumption from the figure (see page 8 of the same reference). It is

required to construct an equilateral triangle on the line AB. Draw a circle with center A and radius AB. Now draw a circle with center B and the same radius, and call C one of the two points where the circles intersect. Then AB = AC since they are radii of the left circle and AB = BC since they are radii of the right circle. But things equal to the same thing are equal to each other, so the triangle is equilateral, *quod erat faciendum.* But Euclid goofed: nothing in his axioms allows us to deduce that the two circles intersect.

Euclidean geometry was made correct only at the end of the 19th century, by the very great German mathematician David Hilbert and, simultaneously and independently, by the Texan mathematician R. L. Moore at the age of nineteen. He looks too handsome to be a mathematician, doesn't he? He was a fine mathematician but a notorious racist. Hilbert, on the other hand, was a feminist and an outspoken critic of anti-Semitism even under the Nazi regime. More honor to him.

The second comment is that until recent times Euclid's geometry was regarded as truth about physical space. One of the first to oppose this notion was Bishop Berkeley in his attacks on the scientism already prevalent in the 18th century. Today the position is untenable on two grounds: the large-scale geometry of spacetime is non-Euclidean, and on the small scale the physical world is quantum mechanical.

Modern mathematics began in 16th century Italy. We all learned in school how to solve quadratic equations by an algebraic formula: if

$$ax^2 + bx + c = 0, \qquad a \neq 0$$

then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

But mathematicians were certain that cubic (third degree) equations could not be solved algebraically, although they had been solved by geometric constructions in eleventh century Persia by Omar Khayyam. I take pleasure in the thought that no pharaoh, no emperor, has a tomb half so magnificent as the tomb of this mathematician. In the West, Omar Khayyam is known primarily as a poet, for his *Rubaiyat*, or collection of quatrains. His most famous quatrain goes something like this:

> *A book of verses under the bough,*
> *A jug of wine, a loaf of bread – and thou,*
> *Sarah, singing in the wilderness –*
> *O, wilderness were paradise enow!*

But I am digressing from mathematics. It was "known" that cubic equations could not be solved algebraically. Amazingly, mathematicians of the Italian Renaissance found such a formula, and even one for fourth-degree equations! It is a lurid tale of passion, intrigue, betrayal, and brilliant insight. It would make a stirring opera: *Solution of the cubic*, featuring as a dramatic duet Ferrari's defeat of Tartaglia in their debate. The protagonist is Girolamo Cardano, whose Ars Magna is the first book of modern mathematics. Cardano is such a fascinating figure that I could easily use up twice my allotted time just talking

about him—but don't worry, I won't. In another book, on games of chance, he invented probability theory long before Pascal and Fermat. In it he states, correctly, that if you have a certain number of equally likely outcomes, some of which are favorable while others are not, the probability of success is the number of favorable outcomes divided by the total number of outcomes. He adds, verbatim, "and nothing else affects the outcome." In the same book he asserts that it is advantageous when playing cards to have the moon shining over your left shoulder. He did not publish this work because at a certain point in his life he was banned from publishing anything by the Inquisition. When this happened Cardano immediately went from Milan to Rome, where with renaissance chutzpah he sought and obtained a papal appointment.

The next century saw the invention of the calculus by Leibniz and Newton. The logical foundations were far from clear, and it was Bishop Berkeley in *The Analyst* who pointed this out with depth and wit. It took two centuries to establish what mathematicians regarded as firm foundations for the calculus.

The early 20th century was a period of intense scrutiny and debate concerning the foundations of mathematics. The subject matter of mathematics is abstract, and therefore the notion of truth *in mathematics* is abstract. I emphasize that I am not saying that historical truth, for example, is abstract—just that the technical notion of truth in mathematics is abstract. Mathematicians and philosophers continue to debate these matters.

But the life blood of mathematics is not truth, but proof, and this has been so ever since the days of Pythagoras and Euclid. What has kept mathematics from fragmenting into competing schools is that proofs are concrete, and mathematicians always, after sufficient study, agree as to whether a proof is correct, whether it accords with the concrete syntactical rules of deduction. This is a great blessing! Position, authority, age, and influence play no role in evaluating mathematical work, and an unknown young person can and frequently does have his or her work recognized very quickly.

In the controversies of the early 20th century, David Hilbert had a wonderful insight. He proposed to set aside the debatable notion of truth in mathematics and instead to prove that mathematics is *consistent.* This is a concrete notion: consistency means that there is no proof both of an assertion and of its negation. His program led to many important discoveries in mathematical logic, but then came the devastating theorem of Kurt Gödel: it is impossible to prove the consistency of mathematics by the methods of mathematics, *if* mathematics is consistent.

But is contemporary mathematics consistent? I took an informal poll of students of foundations, and found that by and large the going odds are a hundred to one. How many of us would board an airplane if we knew that one out of every hundred flights ended in a crash? This is indeed a far cry from the certainty popularly attributed to mathematics.

So mathematicians proceed without certainty, hoping that the foundations are consistent. But this is not the kind of hope that Msgr. Giussani speaks of in the book that we are celebrating today.*

* Luigi Giussani, *Is It Possible to Live This Way? An Unusual Approach to Christian Experience. Vol. 2 Hope*, McGill-Queen's University Press, 2008.

And this brings me back to my starting point. The thrust of this reflection has been where not to look for the kind of certainty, the kind of hope, that transforms lives. Allow me to be so presumptuous as to conclude with

Advice to young cielini

Give no credence to scientism.

Do not look for spiritual insight in the *subject matter* of science and mathematics. Those of us who pursue them are sustained in our daily struggles to bring something new and valuable into being by deep and mysterious sources, deeper than reason, led by a passion for beauty and a glimpse, however partial, of truth. These are noble callings, but they are not the road to ultimate reality.

William Blake understood clearly the relative positions of science and faith:

> *The atoms of Democritus*
> *And Newton's particles of light*
> *Are sands upon the Red Sea shore*
> *Where Israel's tents do shine so bright.*

I honor the soul within you.

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