**Mat104 Fall 2002, Improper Integrals From Old Exams**

For the following integrals, state whether they are convergent or divergent, and give your reasons.

1. $\displaystyle\int_0^\infty \frac{dx}{x^3+2}$
2. $\displaystyle\int_0^1 \frac{dx}{x+\sqrt{x}}$
3. $\displaystyle\int_1^\infty \frac{\sqrt{1+x}}{x^3}$
4. $\displaystyle\int_0^\infty \frac{x^2}{x^3+1}\,dx$
5. $\displaystyle\int_0^1 \ln x\,dx$
6. $\displaystyle\int_0^1 \frac{dx}{e^x-1}$
7. $\displaystyle\int_0^\infty \frac{dx}{x^2+2x+2}$
8. $\displaystyle\int_1^\infty \frac{x^3}{\ln x+x^4}\,dx$
9. $\displaystyle\int_0^\infty \frac{dx}{x^3+\sqrt{x}}$
10. $\displaystyle\int_0^1 \frac{dx}{1-\cos x}$
11. $\displaystyle\int_0^\infty e^{-x}\cos x\,dx$
12. $\displaystyle\int_0^\infty \frac{e^{-x^2}}{x^2}\,dx$
13. $\displaystyle\int_0^\infty \frac{x^2+10}{3x^5+6x+8}\,dx$
14. $\displaystyle\int_0^\infty \frac{x^4+3x+1}{x^5+2x^2+3}\,dx$
15. $\displaystyle\int_0^1 \frac{e^x}{x}\,dx$
16. $\displaystyle\int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$
17. $\displaystyle\int_0^1 \frac{dx}{x^2+\sqrt{x}}$
18. $\displaystyle\int_0^1 (1-x)^{-2/3}\,dx$
19. $\displaystyle\int_2^\infty \frac{x^2+4x+4}{(\sqrt{x}-1)^3\cdot\sqrt{x^3-1}}\,dx$
20. $\displaystyle\int_0^{\pi/2} \tan x\,dx$
21. $\displaystyle\int_0^\infty \frac{e^x-1}{e^{2x}+1}\,dx$
22. $\displaystyle\int_2^\infty \frac{\sin x}{x^2-1}\,dx$
23. $\displaystyle\int_1^\infty \frac{\sin\sqrt{x}}{x+x^4}\,dx$
24. $\displaystyle\int_0^1 \frac{\sin\sqrt{x}}{x+x^4}\,dx$
25. $\displaystyle\int_0^2 \frac{dx}{|x-1|}$
26. $\displaystyle\int_1^\infty \frac{dx}{x^{0.99}}$
27. $\displaystyle\int_0^\infty \frac{dx}{x^4+x^{2/3}}\,dx$
28. $\displaystyle\int_0^\infty x^3\,e^{-x}\,dx$
29. $\displaystyle\int_1^\infty \frac{\ln x}{1+x^2}\,dx$
30. $\displaystyle\int_1^\infty \frac{dx}{x^2\ln x}$
31. $\displaystyle\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}}$
32. $\displaystyle\int_0^\infty e^x(1+e^{-2x})\,dx$
33. $\displaystyle\int_0^1 \sqrt{x}\ln x\,dx$
34. $\displaystyle\int_2^\infty \frac{dx}{x^3-1}$
35. $\displaystyle\int_0^{\pi/2} \frac{1+\cos x}{x}\,dx$
36. $\displaystyle\int_1^\infty \frac{\ln x\cdot\cos x}{x^2+1}\,dx$
37. $\displaystyle\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$
38. $\displaystyle\int_1^\infty \frac{dx}{\sqrt{1+x^4}}$
39. $\displaystyle\int_0^\infty \frac{dx}{\sqrt[3]{x}+x^2}$

**Other problems involving improper integrals**

(1) Find the arc length of the curve given by $x = e^{-t}\cos t$ and $y = e^{-t}\sin t$ for $0 \leq t < \infty$.

(2) Find $\displaystyle\int_0^\infty te^{-t}\,dt$ or show that it diverges.

(3) Evaluate $\displaystyle\int_1^{\sqrt{e}} \frac{\arcsin(\ln x)}{x}\,dx$.

(4) Evaluate $\displaystyle\int_1^\infty \frac{dx}{x^2+1}$.