## Subject Index

## 3K structure, $307,\,308$

abelian varieties, principally polarized, 324-328 angles, 17, 35-37, 55-57, 163, 175, 195, 373-374.405 normalized, 163, 176, 195, 257, 364, 373 approximate identity, 388 Artin-Schreier curves, 301 bi-bounded, see also bounded distortion  $C^1$ -diffeomorphism, 30 homeomorphism, 74, 75, 78, 98 Birch & Swinnerton-Dyer Conjecture, 15 bounded distortion, see bi-bounded  $C^1$ -diffeomorphism of, 30  $\mathcal{C}^1$  partial coordinate system of, 30 characteristic polynomial (reversed), 141 Chebyshev's lemma, 59 compact support modulo the diagonal, 101 "correlation" (Cor), 54 cumulative distribution function, of a Borel measure, 20 of  $\mu(\text{univ}, \text{sep.'s } n)$ , 204 curves Artin-Schreier, see Artin-Schreier curves elliptic, see elliptic curve hyperelliptic, see hyperelliptic curve "cutoff", 77 degree, of a closed point, 271 Deligne's equidistribution theorem, 275, 276, 287Density Lemma, 264 determinant, Fredholm, 142, 211, 235 spectral, 142, 211, 234 Vandermonde, 109 determinant-trace inequality, 156 direct image properties (of spacing measures), 65 - 71Dirichlet L-function, see L-function, Dirichlet discrepancy,  $\varphi$ -discrepancy (of two measures), see measure, discrepancy

distribution function, see cumulative distribution function eigenvalue, Frobenius, see Frobenius eigenvalue eigenvalue location measure, see measure, eigenvalue location elliptic curve, 14, 280, 326, 341 embedding, Segre, see Segre embedding equidistribution theorem, see Deligne's equidistribution theorem Euler-Poincaré formula, 296 everywhere tame, 296 expected value (of a measure), see measure, expected value Fermat curve, 8 finite rank, operator, see operator of finite rank Fredholm determinant, see determinant, Fredholm Frobenius conjugacy class, 5, 268 unitarized, 5 eigenvalue distribution in families of abelian varieties, 365-366 of curves, 364-365 of hypersurfaces, 366, 368-369 of Kloosterman sums, 367 geometric, see geometric Frobenius fundamental group (geometric), 268 Gauss' integral formula, 99 Gauss sum, 7 genus, 304 geometric Frobenius, 267 fundamental group, see fundamental group (geometric) generic fibre, 283 monodromy group, see monodromy group point, 267 M- $\varphi$ -grid diameter, 78, 85 M- $\varphi$ -grid discrepancy, 80

grid points, 78 finite, 78 "grid size", 77 "group size", 77 GUE measure, see measure, GUE GUE discrepancy in families, 352 of abelian varieties, 355 of curves. 353 of hypersurfaces, 356 of Kloosterman sums, 358 Hadamard's lemma, 398 Heine's formula, 391 hyperelliptic curve, 7, 13, 293, 300, 302, 311, 320, 365 hyperelliptic probability measure, see measure, hyperelliptic probability inertia group, 297 "integral" (Int), 54 intersection multiplicity, 305 Kloosterman sum, 6, 348 Kolmogoroff-Smirnov discrepancy, see measure, discrepancy L-function, Dirichlet, 13 Lebesgue - Stieltjes positivity condition, 191 Lefschetz pencils, 312 level n structure, 305 Limit lemma, 191 lisse, sheaf, see sheaf, lisse literal spacings, see vector, of spacings, literallow lying zeros, see zeros, low lying mass formula, 309, 314, 329, 346 measure. calculate strongly, 12 calculate weakly, 12 classical spacing, 208 discrepancy, 3, 20  $\varphi$ -discrepancy, 75, 79 eigenvalue location, 378-381 expected value of, 37, 57 exponential decay, 221 GUE, 1, 2 hyperelliptic probability, 321intrinsic probability, on  $USp(2g)^{\#}$ , 316, 330 multi-eigenvalue location, 176 naive probability, 329 Poisson, 9 spacing, 23 moments of offset, 194-195 offset, 185-189 static spacing, 177, 181

tail of, 189–192

universal probability, 24 Wigner, 2 modular curve. 8 moduli problem, 324–327 Moebius Inversion Relation, 192 monodromy group, 283 monodromy of families, 293 of curves of genus g > 2, 304–307 of hyperelliptic curves, 320-321 of hypersurfaces, 331-334 of Kloosterman sums, 347-349 of principally polarized abelian varieties, 323-324 Montgomery-Odlyzko Law, 1, 3 Mordell-Weil group, 15 multi-eigenvalue location measure, see measure, multi-eigenvalue location naive probability measure, see measure, naive probability naive spacings, see vector, of spacings, naive normalized normalized spacings, see vector, of spacings, normalized offsets, see vector, of offset operators, of finite rank, 141 integral, 141-143 "order" map, 51 Poisson distribution, 9 Poisson measure, see measure, Poisson polynomial, characteristic, see characteristic polynomial positive trace class, 234 principal polarization of A/S, 323 isomorphism classes, 355 principally polarized abelian varities, see abelian varieties, principally polarized projective automorphism, 335 pure (lisse sheaf), 268 ι-pure (lisse sheaf), 268  $\varphi$ -rectangle, marked, 78 semi-finite, 74, 78 semi-infinite, 74, 78 "reversing" map, 39 Riemann hypothesis for abelian varieties, 329 for C/k, 315 for curves over finite fields, 1, 295, for  $L(s, \chi_K)$ , 13 for  $Y^2 = f(X)$ , 299 Sato-Tate conjecture, 7 for  $X_0(l)$ , 8, 9

426

```
scaling limits, 212-215, 251-263, 385
Segre embedding, 332
Serre's lemma, 325
sheaf, 268
  lisse, 268
sine ratios, 118–120
spacing measure, see measure, spacing
spacing vector see vector, of spacings
spacings,
  literal, 17
  normalized, 2, 17, 30, 183
Tail estimate, 174, 175
Tail integration lemma, 170
tame, see everywhere tame
Teichmuller decomposition, 285
"total correlation" (TCor), 54
universal probability measure, see measure,
    universal
probability
Vandermonde determinant, see determinant,
    Vandermonde
varieties, see abelian varieties
vector,
  of offsets, 18, 46, 50, 186, 192, 195, 378,
    401
  of separations, 18, 46, 50, 186
  of spacings,
    literal, 18
    naive normalized, 35-37
    normalized, 18
  of steps, 18, 186
  Witt, 344
Weyl integration formulas, 107-109
  K_N(x,y) versions, 109–116
  explicit, via S_N, 120–121
Widom's theorem, 385
Wigner measure, see measure, Wigner
Wigner surmise (unitary), 412
zeros,
  low lying, 9
    of elliptic curves, 14
    of Dirichlet L-series, 13
zeta function,
  of A/k, 329, 355
  of C/F_q, 1, 4, 306, 331, 353
  of C/k, 315
```