Name ______________________________ **Class Time** ______________

MATH 104 - QUIZ # 4
Spring 2003
Due Monday, April 28 at 2PM
On Alternating Series, Taylor Series and Power Series
Time: 60 minutes

Please show all work. Books, notes, calculators, are not permitted on this quiz. As part of your obligations under the Honor Code, do not discuss this quiz with anyone until after the Monday 2PM deadline.

WRITE OUT AND SIGN THE PLEDGE:
I pledge my honor that I have not violated the Honor Code during this examination.

1. *(12 points)* For each of the series below, write **AC** if the series converges absolutely, **CC** if the series converges conditionally and **D** if the series diverges. *Please circle your answer* and briefly justify it — that is, tell which tests you are using to back up your conclusions. (You do not need to work out all the details explicitly.)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{(\ln 2)^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n)!}{5^n \cdot n! \cdot n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}(\sqrt[n]{e} - 1)$

2. *(10 points)* Find the second-order Taylor polynomial of the function $f(x) = \sqrt[3]{x}$ near $x = 8$. Compute the approximate value of $\sqrt[3]{10}$ given by this polynomial. (Leave your answer in fraction form, but simplify it.)

3. *(8 points)* For which values of x does the power series $\displaystyle\sum_{n=2}^{\infty} \frac{(x-4)^n}{n \ln^2 n}$ converge? Justify your answer.

4. *(10 points)*

(a) State (or compute) the Taylor series centered at 0 of $f(x) = \ln(1 + x)$.

(b) State (or compute) the Taylor series centered at 0 of $g(x) = \dfrac{1}{1 + x^2}$.

(c) Compute the first four nonzero terms of the Taylor series centered at 0 of $h(x) = \dfrac{\ln(1 + x)}{1 + x^2}$.

5. *(10 points)* Compute the following limits:

(a) $\displaystyle\lim_{x\to 0} \frac{\sin 3x(1-\cos 2x)}{e^{x^3}-1}$

(b) $\displaystyle\lim_{x\to\infty} x^2\left(e^{-1/x^2}-1\right)$