

Mathematics 104
Fall Term 2003-2004
Final Examination
January 16, 2004

1. Evaluate $\int \frac{5 dx}{x^3 + 2x^2 + 5x}$.

2. For each of the following integrals, state whether it converges or diverges, and give your reasons carefully and clearly.

a. $\int_{-\infty}^{\infty} \cos 2t dt$.

b. $\int_1^{\infty} \frac{x^3 dx}{1 + x^4}$.

3. For each of the following series, state whether it converges or diverges, and give your reasons carefully and clearly.

a. $\sum_{n=1}^{\infty} e^{-n \ln n}$.

b. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{1 + \frac{1}{n}}$.

4. Find the Taylor series, centered at -1 , of $f(x) = \frac{1}{x}$.

5. Estimate $\int_0^{1/2} e^{-x^3} dx$ with an error no bigger than $1/100$. Give your reasons.

6. Find $\lim_{x \rightarrow 0} \frac{(x - \sin x)^2}{x^6}$.

7. Find the area between the origin and the curve given in polar coordinates by $r = \theta e^\theta$ for $0 \leq \theta \leq \pi$.

8. Find all roots of $x^6 - 3x^3 + 9 = 0$ in polar form: $x = r e^{i\theta}$.

9. Consider the region under the curve $y = e^{-x}$ and above the x -axis for $0 \leq x < \infty$.

a. Revolve it around the x -axis and find the volume.

b. Revolve it around the y -axis and find the volume.

10. Find the arc length of the curve given by $y = x^2$ for $0 \leq x \leq \sqrt{2}$. (You may find the formula

$$\int \sec^3 \theta d\theta = \frac{\sec \theta \tan \theta}{2} + \frac{1}{2} \int \sec \theta d\theta$$

useful.)

11. The mass m of a crystal in a solution grows at a rate *proportional* to $m^{2/3}$. The original mass is 1 gram and the mass after 24 hours is 8 grams. Find the exact value of the mass as a function of time.