(1) Since $e^{nx} = (e^x)^n$ this is a geometric series with $r = e^x$. It converges absolutely, provided $|e^x| < 1$, that is for $x \in (-\infty, 0)$. In that case, it will converge to $\frac{1}{1 - e^x}$.

(2) This is a geometric series with $a = (-2/3)^4$ and $r = -2/3$. Therefore it converges to $\frac{(-2/3)^4}{1 + 2/3} = \frac{16}{135}$.

(3) Here we combine several geometric series:

\[ \sum_{n=0}^{\infty} \frac{2^n}{5^n} = \frac{1}{1 - 2/5} = \frac{5}{3} \quad \sum_{n=0}^{\infty} \frac{3^{n+1}}{5^n} = \frac{3}{1 - 3/5} = \frac{15}{2} \quad \sum_{n=0}^{\infty} \frac{4^{n+2}}{5^n} = \frac{16}{1 - 4/5} = 80 \]

the series we are given will converge to $5/3 + 15/2 + 80 = \ldots$.

(4) Answer: $2 + 1/2 - 3/8 = 17/8$ (similar to problems 1-3 above)

(5) Answer: $8/3 + 2 = 14/3$. (similar to problems 1-3 above)

(6) As $n \to \infty$, both the numerator and the denominator go to infinity. Thus we can use L’Hôpital’s Rule:

\[ \lim_{n \to \infty} \frac{\ln(n^2 + n)}{\ln(n^2 - n)} = \lim_{n \to \infty} \frac{2n + 1}{n^2 + n} = \lim_{n \to \infty} \frac{2n + 1}{2n - 1} \cdot \frac{n^2 - n}{n^2 + n} = 1 \]

since the leading term on top and bottom is now $2n^3$.

(7) This limit will be 0. If we apply L’Hôpital’s Rule repeatedly we end up with $\frac{18}{24n}$ which goes to 0 as $n$ goes to infinity.

(8) This limit exists and equals $-1$. (Recall arctan $n$ goes to $\pi/2$ as $n$ goes to infinity.)

(9) Since $(1 + 1/n)^n$ goes to $e$ as $n$ goes to infinity, this will go to $e^2$.

(10) (a) The numerator goes to $e$ and the denominator goes to $\infty$. So the quotient will go to 0 as $n$ goes to $\infty$.

(b) An $\frac{\infty}{\infty}$ form so use L’Hôpital’s Rule to show that the limit is $1/2$. 
