Mat104 Problems on Complex Numbers From Old Exams

(1) Find all solutions of $z^5 = 6i$.

(2) Find the real part of $(\cos 0.7 + i \sin 0.7)^{53}$.

(3) Find all complex numbers z, in Cartesian (rectangular) form such that $(z-1)^4 = -1$.

(4) Write $(\sqrt{3} + i)^{50}$ in polar and in Cartesian form.

(5) Find all fifth roots of -32.

(6) Write the following in Cartesian form $a + ib$ where a and b are real and simplified as much as possible:

(a) $\dfrac{1}{1+i} + \dfrac{1}{1-i}$ (b) $e^{2+i\pi/3}$

(7) Write all solutions of $z^3 = 8i$ in polar and Cartesian form, simplified as much as possible.

(8) Find all complex solutions of the equation $z^5 = 1 + i$.

(9) Find the imaginary part of $\dfrac{2+i}{3-i}$.

(10) Find the angle between 0 and 2π that is an argument of $(1-i)^{1999}$.

(11) Find all z such that $e^{iz} = 3i$.

(12) Write $(1-i)^{100}$ as $a + ib$ with a and b real numbers and simplify your answer.

(13) Find the real part of $e^{(5+12i)x}$ where x is real, and simplify your answer.

(14) Find all solutions to $z^6 = 8$ and plot them in the complex plane.

(15) Evaluate $\displaystyle\sum_{n=0}^{\infty} \frac{\sin n\theta}{n!}$.

(16) For what θ does $\displaystyle\sum_{n=0}^{\infty} \frac{\cos n\theta}{2^n}$ converge? If it converges, what does it converge to?