Mathematics 104
Spring Term 2006
Final Examination
May 18, 2006

1. Evaluate $\displaystyle\int \frac{x^2}{(1+x^2)^{3/2}}\, dx$.

2. Evaluate $\displaystyle\int \frac{\ln(x^2 + 2x + 2)}{(x + 1)^2}\, dx$.

3. Does $\displaystyle\int_2^\infty \frac{\ln(e^x - 2)}{x^3 + 1}\, dx$ converge or diverge? Give your reasons.

4. (a) Does $\displaystyle\sum_{n=0}^\infty \frac{3^n (n!)^2}{(2n)!}$ converge or diverge? Give your reasons.

   (b) Does $\displaystyle\sum_{n=1}^\infty \frac{e^{10n} + n^{10}}{n^n}$ converge or diverge? Give your reasons.

5. For what values of x does $\displaystyle\sum_{n=2}^\infty \frac{x^n}{n(\ln n)^{\frac{1}{2}}}$ converge? Give your reasons.

6. Find $\displaystyle\lim_{x\to 0} \frac{e^{2x} - \cos x - \sin 2x}{\ln(1 + x) - x}$. Show your work.

7. Write $(1 + i)^{15}(1 + i\sqrt{3})^{17}$ in polar form $re^{i\theta}$ with $r \geq 0$ and $0 \leq \theta < 2\pi$.

8. Find all real solutions to the differential equation $\cos^2 x\dfrac{dy}{dx} + y = e^{\tan x}$. Show your work.

9. Find all real solutions to the differential equation $\dfrac{d^2 y}{dx^2} + \dfrac{dy}{dx} - 2y = e^{3x}$. Show your work.

10. Find the volume of the solid obtained by revolving the region under the curve $y = \cos x$ and above the x-axis, for $0 \leq x \leq \pi/3$, about the line $x = -1$. Show your work.

11. Find the length of the curve given in parametric form by

$$\begin{cases} x = 2(t^2 - 1)^{3/2} \\ y = 3t^2 \end{cases}$$

where $2 \leq t \leq 3$.