Note: Average was approximately 60 percent. Considered hard but fair.

1. (8 points) Compute the following integrals:
   (a) \( \int \frac{x}{(1 - x^2)^{3/2}} \, dx \)
   (b) \( \int x \ln(x + 1) \, dx \)

2. (12 points)
   (a) Let \( R \) be the region bounded by the \( x \)-axis and the graph of \( y = 1/(x^4 + 1) \) as \( x \) runs from 0 to \( \infty \). Find the volume of the solid of revolution obtained by revolving \( R \) about the \( y \)-axis.
   (b) Calculate the area of the surface obtained by revolving the graph of \( y = e^x \) between the points \((0, 1)\) and \((1, e)\) around the \( x \)-axis.

3. (16 points) Determine whether the following integrals converge or diverge. Give your reasons.
   (a) \( \int_0^\infty \frac{dx}{\sqrt{x} + x^3} \)
   (b) \( \int_0^1 \frac{\tan \sqrt{x}}{x + x^2} \, dx \)
   (c) \( \int_0^1 \frac{\ln(1 + x)}{x^3} \, dx \)
   (d) \( \int_1^\infty \frac{dx}{x \ln x} \)

4. (16 points) Determine whether the following series converge or diverge. Give your reasons.
   (a) \( \sum_{n=0}^{\infty} \frac{n^2}{\sqrt{n^6 + 1}} \)
   (b) \( \sum_{n=0}^{\infty} (-1)^n n^2 \frac{n^2 + 1}{n!} \)
   (c) \( \sum_{n=0}^{\infty} \frac{n^2 + 3^n}{n!} \)
   (d) \( \sum_{n=0}^{\infty} \frac{(n + 1)^n}{n + 3} \)
5. (12 points) Let \( f(x) = \sum_{n=0}^{\infty} \frac{1}{n+2} \left( \frac{x-2}{3} \right)^n \).

(a) For what values of \( x \) does the series converge? Give your reasons.
(b) Find \( f^{(50)}(2) \).

6. (12 points)

(a) Use Taylor series to compute \( \lim_{x \to 0} \frac{(e^x - 1 - x)^2 \cos x}{x(\sin x - x)} \).

(b) Find the Taylor series of \( F(x) = \int_0^x \frac{dt}{1 + t^4} \) centered at \( x = 0 \). For what values of \( x \) does it converge?

7. (12 points) For the questions below express your answers in the form \( a + ib \) where \( a \) and \( b \) are real numbers. Simplify your expressions for \( a \) and \( b \).

(a) Simplify \( \left( \frac{7 + i}{3 + 4i} \right)^{43} \).

(b) Solve \( z^4 = -8iz \).

8. (12 points) Find all real solutions to the following differential equations.

(a) \( y'' + 2y' + 10y = 0 \)

(b) \( 2y'' + y' - 3y = 0 \)