Introduction to abstract algebra with applications to social systems

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In recent years, with the rise in popularity of social networking websites such as Facebook, there has been considerable new interest in the science and mathematics of social networks. In fact, networks are found all around us. For example, consider airplane flight patterns for a given airline or the spread of disease through a population.

In the first half of the course, we will explore how we can use matrix algebra to represent interactions within a network of individuals. By first illustrating relationships in a diagram called a *graph*, we can then translate the interactions into a matrix, which we can manipulate algebraically to answer questions about each individual's position in the network. We will also discuss how to partition a large group of individuals into subsets (called *equivalence classes*) in which all individuals in a given subset perform the same role in the network.
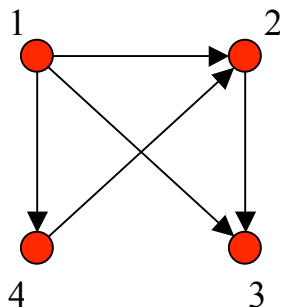

Figure 1. Example network graph

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Figure 2. Adjacency matrix for example network

In the second half of the course, we will explore one of the key concepts in abstract algebra – the concept of a *group*. (Note that this algebraic definition will be different from the sociological definition of group as "a collection of individuals.")   To illustrate the mathematical concept of a group, we will investigate one of the earliest examples of a social network—marriage rules in primitive societies.

In some primitive societies, there are rigid rules as to when marriages are permissible. These rules are designed to prevent very close relatives from marrying. The rules can be given precise mathematical formulations in terms of permutation matrices. By visualizing relationships between individuals in diagrams called family trees, we will be able to represent all possible male-female familial relationships by just a few matrices. These matrices will constitute a group. We will explore the properties of groups, which will allow us to conclude with a description of the formal marriage structure for a number of primitive societies.