Conference in Memory of John N. Mather

October 1 – 3, 2018

Taplin Auditorium, Princeton University

Speaker: Jim Damon

Title: John Mather's Pioneering Work in Singularity Theory and Its Enduring Legacy

Abstract:

The talk will concentrate on John Mathers work on smooth equivalence of singularities, including infinitesimal stability, his main contributions to the local theory of singularities and his determination of the "nice dimensions". The historical setting for John Mathers work, resulting from earlier work of Thom, Whitney and Malgrange, will be described to provide a background for the dramatic advances his work provided.

Speaker: Albert Fathi

Title: Recurrence on abelian cover. Application to closed geodesics in manifolds of negative curvature

Abstract:

If h is a homeomorphism on a compact manifold which is chain-recurrent, we will try to understand when the lift of h to an abelian cover (i.e. the covering whose Galois group is the first homology group of the manifold) is also chain-recurrent.

This is related to the proof by John Franks of the Poincaré-Birkhoff theorem.

It has consequences on density of classes of closed geodesics in a manifold of negative curvature, and more generally on geodesic flows which are Anosov.

Speaker: Charles Fefferman

Title: Systems of Equations for C^m Functions

Abstract:

The talk treats linear equations

$$\sum_{j=1}^{J} A_{ij}(x)F_j(x) = f_i(x) \qquad (i = 1, \ldots, I)$$

for unknown C^m functions $F_1, \ldots, F_J$ on $\mathbb{R}^n$, with m fixed.

1. Suppose the A_{ij} and f_i are given. How can we tell whether a C^m solution exists?
2. Suppose we restrict the A_{ij} and f_i to be polynomials. The vectors $(f_1, \ldots, f_I)$ for which a C^m solution exists form a module over the ring of polynomials on $\mathbb{R}^n$. How can we exhibit generators, and what can we say about the solutions $F_1, \ldots, F_J$?

Joint work with János Kollár and with G.K. Luli.

Speaker: Jacques Féjoz

Title: From finite-time unbounded singularities, to the question of stability in the N-body problem

Abstract:

In 1975, J. Mather and R. McGehee showed for the collinear four-body problem that a Cantor set of initial conditions lead to unbounded solutions in physical space, in a finite time — a then new type of singularity of solutions of the N-body problem. I will describe this beautiful work, and discuss some later subjects of interest of J. Mather in celestial mechanics, related to questions of stability.

Speaker: Giovanni Forni
Title: On twisted ergodic integrals
Abstract:
We discuss results on effective equidistribution of the products of horocycle flows, their time-changes and translation flows times a linear flow on a circle. The results are related to the mixing and spectral properties of the flow themselves. For horocycle flow we draw from work of Venkatesh, and from joint work with Ulcigrai and with Flaminio and Tanis. For translation flows we introduce a geometric, cohomological version of the Bufetov-Solomyak cocycle and prove an upper bound on its Lyapunov exponents.

Speaker: Mark Goresky
Title: Mather's work on stratification theory and topological stability.
Abstract:
John Mather, following an outline proposed by R. Thom and earlier efforts of H. Whitney, made stratification theory into a powerful tool with applications to a wide range of mathematical phenomena. I will give a somewhat historical survey of this development and several applications of the theory.

The talk will also briefly explain how the local results of John Mather extend to a more general form of "Thom-Mather Theory". We will point to relations to other fundamental methods, and explain its general applicability to many other theoretical situations not treated by John Mather. Along with this will be a brief sample of various applications of this general theory from among: bifurcation theory, solutions to PDEs, generic differential geometry, and computer imaging.

Speaker: Vadim Kaloshin
Title: Mather's program of proving Arnold diffusion for nearly integrable Tonelli Hamiltonians.
Abstract:
During the talk we present Mather's theory of minimal measures for Tonelli Hamiltonians. In the late 90s–early 2000s using this theory Mather proposed a program of constructing instabilities for near integrable systems and proving existence of Arnold diffusion. Recently some steps of this program were realized.

Speaker: Leonid Polterovich
Title: Persistence barcodes in analysis and geometry
Abstract:
While originated in topological data analysis, persistence modules and their barcodes provide an efficient way to book-keep homological information contained in Morse and Floer theories. I shall describe applications of persistence barcodes to symplectic topology and geometry of Laplace eigenfunctions. Based on joint works with Iosif Polterovich, Egor Shelukhin and Vukasin Stojisavljevic.

Speaker: Alfonso Sorrentino
Title: Aubry-Mather theory: the principle of least action in dynamics and beyond
Abstract:
This talk will focus on John Mather's seminal contributions to the study of the dynamics of monotone twist maps of the annulus and Tonelli Hamiltonian systems, by means of the so-called "Principle of Least Action". This variational principle roughly states that, for sufficiently short times, trajectories of these dynamical systems minimize the so-called Lagrangian action, among all paths in configuration space with the same end points.

In the '80s, Serge Aubry and John Mather proved, independently, that monotone twist maps on an annulus possess global minimizers of the action for all rotation numbers, and thoroughly describe their dynamics and their structure. Later, these results where extended by John to Hamiltonian systems in higher dimension. These minimizing orbits turned out to be of crucial importance for a deeper understanding of the complicated dynamics of these systems and the onset of chaos in classical mechanics.

In this talk I shall review the basics of this theory — that nowadays is commonly called Aubry-Mather theory — and describe how these sets of ideas turned out to have significant connections to many other problems in PDE, billiard dynamics, symplectic geometry, etc...

Speaker: Dennis Sullivan
Title: Finite Dimensional Models for Incompressible Fluids in Periodic Three Space
Abstract:
This reports on future plans for work in progress with Theodore Drivas of Princeton Daniel An of Maritime College and Pooja Rao, Phd Applied Math Stony Brook. The first model is derived from the PDE for incompressible fluids and lives on finite sums of eigenspaces of the curl operator. It is an ODE which preserves the euclidean measure, the energy norm, and ignoring the laplacian term, "transports" the curl of velocity. These properties characterize this model in an appropriate sense. The goal is to understand this model by simulation to predict the theory. Then see if this "curl model" helps with the PDE theory which it approximates.

The second model seems to blow up in finite time unless the friction term dominates [calculations of Daniel An]. The blowup seems like a definite phase change. This lattice model unlike the curl model has no known algebraic properties except incompressibility. However it is derived as is the PDE model using the same truism about momentum. This derivation, with the caveat that the calculus limit is not taken but is replaced by a cumulant discard expressed in the language of combinatorial topology on a multiply layered cubical complex of periodic three space. At the scale where the cumulant is zero this model seems true on physical grounds.