

In 1978 De Giorgi made a conjecture about the symmetry of global solutions to a certain semilinear elliptic equation. He stated that monotone, bounded solutions of

$$\Delta u = u^3 - u$$

in \mathbb{R}^n are one dimensional (i.e. the level sets of u are hyperplanes) at least in dimension $n \leq 8$. This problem is in fact closely related to the theory of minimal surfaces and it is sometimes referred to as "the ε version of the Bernstein problem for minimal graphs". In my talk I will explain this relation and I will give an idea about the proof of this conjecture for $n \leq 8$. We mention that recently Del Pino, Kowalzyk and Wei provided a counterexample in dimension $n \geq 9$.