

CREMONA TRANSFORMATIONS AND
HOMEOMORPHISMS OF TOPOLOGICAL SURFACES

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The simplest Cremona transformation of projective 3-space is the involution

$$\sigma : (x_0 : x_1 : x_2 : x_3) \mapsto \left(\frac{1}{x_0} : \frac{1}{x_1} : \frac{1}{x_2} : \frac{1}{x_3} \right),$$

which is a homeomorphism outside the “coordinate tetrahedron” ($x_0x_1x_2x_3 = 0$).

By studying the action of σ on real quadric surfaces, we show that σ and its conjugates generate a dense subgroup of $\text{Homeo}(S^2)$, the group of homeomorphisms of the 2-sphere.

Then we show that the same holds if the 2-sphere is replaced by the torus or by any non-orientable surface and explain why there can not be similar results for orientable surfaces of genus ≥ 2 .

(Joint work with Frédéric Mangolte.)