Integral zeta values and the number of automorphic representations

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Let $\zeta^*(s) := (1-2^{1-s})\zeta(s)(= 1 - \frac{1}{2^s} + \frac{1}{3^s} - \frac{1}{4^s} + \ldots$ for $\mathrm{Re}(s) > 1)$. Euler proved that the values of $\zeta^*(s)$ at negative integers are elements of the ring $\mathbb{Z}[1/2]$. Cassou-Nogues and Deligne/Ribet generalized this to an integrality result for the values of arbitrary partial zeta functions at negative integers. I will review their results, and show how these special values can be used to compute the number of irreducible automorphic representations of G with prescribed local behavior, where G is a simple group over a global field k. Via the global Langlands correspondence for $k = F(t)$, I will compare this result with work of Katz and Deligne on Kloosterman sheaves. This is joint work with Mark Reeder.