Counting Faces of Randomly-Projected Polytopes, with applications to Compressed Sensing, Error-Correcting Codes, and Statistical Data Mining.

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There are three families of regular polytopes in high dimensions: simplices, hypercubes, and cross-polytopes. Randomly project one of these objects into a lower dimension, and some surprises await. Moreover, these surprises have numerous real-world implications in statistics and signal processing.

Let N denote the dimension of the polytope Q we start with, and n the dimension after projection. Let A denote an $n \times N$ random matrix with iid Gaussian entries; let P be the image of the polytope Q under projection, i.e $P := AQ$.

There is a critical ratio ρ depending on n/N and the type of polytope Q, such that, for $k < \rho(n/N; Q)n(1 + o(1))$ the number of k-dimensional faces of P is the same as the number of k-dimensional faces of Q.

We hope to explain how to calculate ρ for each of the three families of regular polytopes, and we hope to show how these results relate to the current enormous interest in sparse signal representation, including compressed sensing and the fitting of linear models in statistics when there are many more predictors than observations.

This is joint work with Jared Tanner, University of Edinburgh.