The hypoelliptic Dirac operator

If X is a compact Riemannian manifold, the Dirac operator is a first order elliptic self-adjoint operator acting on smooth sections of twisted spinors over X. When X is complex and Kähler, an example of such an operator is given by $\overline{\partial}^X + \overline{\partial}^{X*}$.

The hypoeliptic Dirac operator D_b^X is a first order hypoelliptic operator acting on the total space $\mathcal{X}$ of the tangent bundle of X, which is self-adjoint with respect to a Hermitian form of signature (∞, ∞). The principal symbol of its square is a weighted sum of the harmonic oscillator along the fibre and of the generator of the geodesic flow. As $b \to 0$, D_b^X 'collapses' to D^X. As $b \to +\infty$, $D^{X,2}$ converges in the proper sense to the generator of the geodesic flow.

The hypoelliptic deformation preserves certain fundamental quantities like the Quillen metric of the determinant of the cohomology, at least of up to local terms. In more rigid situations, the deformation is essentially isospectral.